

# Inflation and classical scale invariance



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based on 1410.xxxx  
in collaboration with

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# Summary

## Introduction

- Experimental Data
- Inflation Recipe
- Classical scale invariance
- Theoretical framework

## Model

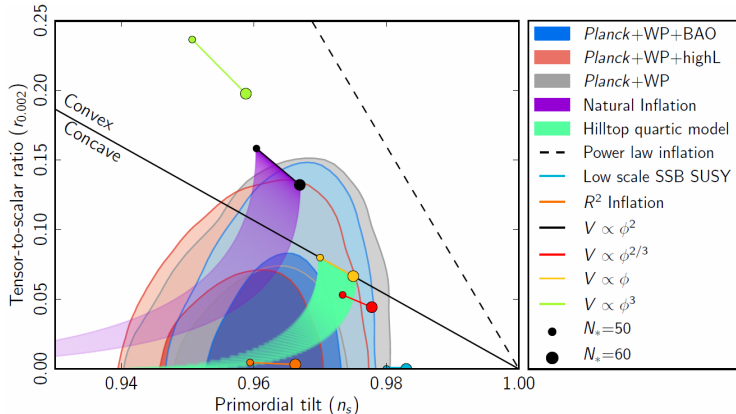
- Preliminaries
- Multiple Point Criticality Principle
- RGEs
- Study of  $U(\phi)$

## Results

- Results I
- Results II
- Results III

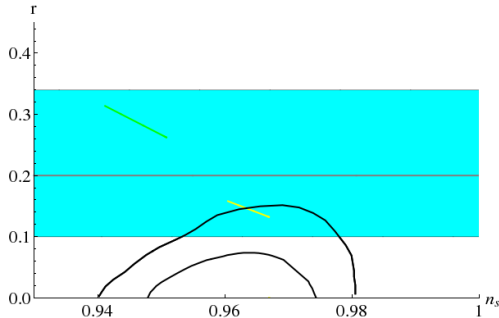
## Conclusions

# Experimental Data: 2013



$$r = \frac{P_T(k)}{P_S(k)} \quad P_S(k) = A_s \left( \frac{k}{k_*} \right)^{n_s - 1 + \dots}$$

# Experimental Data: 2014



$$\mathbf{r} = \frac{P_T(k)}{P_S(k)}$$

$$P_S(k) \sim k^{n_s-1}$$

— Planck bound

■ BICEP2  $2\sigma$  region

—  $V = m^2 \phi^2$

—  $V = \lambda \phi^4$

# Experimental Data and Analysis: 2014

Reactions to  $r = 0.2^{+0.07}_{-0.05}$  (BICEP2 Coll., Phys. Rev. Lett. **112** (2014) 241101)

- ▶ initially: many many papers trying to explain/predict BICEP2 signal
- ▶ later: all the signal could be just dust (or not)
  - M. J. Mortonson and U. Seljak, arXiv:1405.5857
  - R. Flauger, J. C. Hill and D. N. Spergel, JCAP **1408** (2014) 039

⇒ we need more data from Planck

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- a. W. N. Colley and J. R. Gott, [arXiv:1409.4491](#)

The main uncertainty seems to be the amplitude of the dust signal in the BICEP2 map. The previous studies have considered the power spectrum of the B-modes. But this leaves out the other information in the maps. Looking at the power spectrum alone is insufficient.

Therefore they designed a study which does not depend on the amplitude of the dust signal at all (for more details check the article).

$r = 0.11 \pm 0.04$  (based on **preliminary** public Planck dust data)

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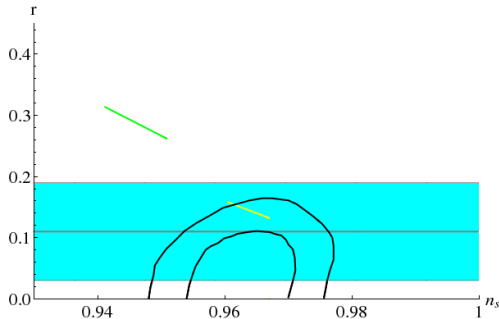
- ▶ Planck collaboration **official** dust data (arXiv:1409.5738)





# Experimental Data and Analysis: 2014

However a.(■) and b.(-) are compatible results.



It is too soon for taking final conclusion.

Hopefully Gert will give soon a seminar on this topic... 😊

# Recipe for studying inflation

$$S = \int d^4x \sqrt{-g} \left( \frac{M_P^2}{2} R + \mathcal{L}_\phi \right) \quad \mathcal{L}_\phi = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi)$$

1. Compute the slow roll parameters

$$\epsilon(\phi) = \frac{M_P^2}{2} \left( \frac{V'(\phi)}{V(\phi)} \right)^2$$

$$\eta(\phi) = M_P^2 \frac{V''(\phi)}{V(\phi)}$$

Inflation takes place when  $\epsilon, \eta \ll 1$

2. Compute the field value at the end of inflation  $\phi_e$  from

$$\epsilon(\phi_e) = 1$$

# Recipe for studying inflation

3. Compute  $\phi^*$ ,  $N$  e-folds before the end of inflation,

$$N = \frac{1}{M_{\text{P}}} \int_{\phi_e}^{\phi^*} \frac{d\phi'}{\sqrt{2\epsilon(\phi')}}$$

4. Compute  $r$  and  $n_s$  at  $N = [50, 60]$  by

$$\begin{aligned} r &= 16\epsilon(\phi^*) \\ n_s &= 1 - 6\epsilon(\phi^*) + 2\eta(\phi^*) \end{aligned}$$

5. From the scalar amplitude measurement:  $A_s^2 = \frac{V(\phi^*)}{24\pi^2 M_{\text{P}}^4 \epsilon(\phi^*)} \approx 2.45 \times 10^{-9}$   
fix the overall normalization of  $V$ :

$$V(\phi^*) \simeq \left(1.94 \times 10^{16} \text{ GeV}\right)^4 \frac{r}{0.12}$$

# Classical scale invariance

- ▶ I do not want to discuss why to choose classical scale invariance, its meaning, pros and cons
- ▶ Let us just treat it as a possible configuration of the parameters space
- ▶ We consider a configuration in which the classical Lagrangian has all the dimensionful parameters set to be zero
- ▶ Therefore mass terms must be generated at quantum level

# Dimensional transmutation

Tree level:  $V_\phi = \frac{1}{4} \lambda_\phi \phi^4 \Rightarrow V_\phi^{\min} : \langle \phi \rangle = v_\phi = 0 \text{ \& } m_\phi^2 = 0$

RGE :  $\beta_{\lambda_\phi} = \frac{d\lambda_\phi}{d \ln \mu}$

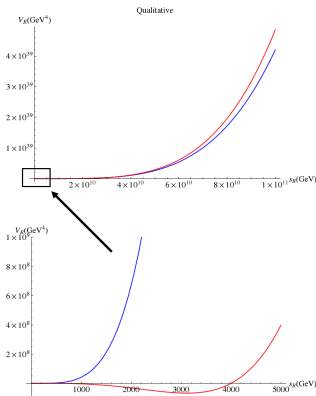
$\Rightarrow$  if:  $\beta_{\lambda_\phi} \sim \text{const.}$  (at least in some region) and  $> 0$

$\Rightarrow \int_{\lambda_{\phi_0}}^{\lambda_\phi} d\lambda_\phi \simeq \beta_{\lambda_\phi} \int_{\ln \mu_0}^{\ln \mu} d(\ln \mu)$

$\Rightarrow \lambda_\phi \simeq \lambda_{\phi_0} + \beta_{\lambda_\phi} \ln \frac{\mu}{\mu_0}$

$\Rightarrow$  fixing  $\lambda_{\phi_0} = 0$

$\Rightarrow \boxed{\lambda_\phi \simeq \beta_{\lambda_\phi} \ln \frac{\mu}{\mu_0}} \rightarrow \begin{cases} \lambda_\phi(\mu > \mu_0) > 0 \\ \lambda_\phi(\mu < \mu_0) < 0 \end{cases}$



One loop:  $V_{R,1\text{loop}} \simeq \frac{1}{4} \beta_{\lambda_\phi} \ln \frac{|\phi|}{\phi_0} \phi^4 \Rightarrow v_\phi \simeq \frac{\phi_0}{e^{1/4}} \text{ \& } m_\phi^2 = \frac{\beta_{\lambda_\phi} \phi_0^2}{\sqrt{e}} (\phi_0 \leftrightarrow \mu_0)$

# Coleman-Weinberg inflation

- ▶ Already the first papers on inflation considered CW inflation:

Linde, Phys. Lett. B108 (1982) 389-393, Phys. Lett. B114 (1982) 431; Albrecht and Steinhardt, Phys. Rev. Lett. 48 (1982) 1220-1223; Ellis et al., Nucl. Phys. B221 (1983) 524, Phys. Lett. B120 (1983) 331.

- ▶ This idea has been (is being) extensively studied in the context of

- GUT:

Langbeine et al, Mod.Phys.Lett. A11 (1996) 631-646; Gonzalez-Diaz, Phys.Lett. B176 (1986) 29-32; Yokoyama, Phys.Rev. D59 (1999) 107303; Rehman et al., Phys.Rev. D78 (2008) 123516.

- $U(1)_{B-L}$ :

Barenboim et al., Phys.Lett. B730 (2014) 81-88; N. Okada and Q. Shafi, 1311.0921.

- $SU(N)$ :

Elizalde et al., 1408.1285.

- ▶ They all suppose new gauge groups beyond the SM: NO NEED FOR IT!
- ▶ It can occur just due to running of some scalar quartic coupling, to negative values at some energy scale due to couplings to other scalar fields, generating non-trivial physical potentials.

# Lagrangian

$$S = \int d^4x \sqrt{-g} \left[ f(\phi) R + \mathcal{L}_{\text{matter}} \right] \quad f(\phi) = \frac{\xi_\phi}{2} \phi^2$$

$$\mathcal{L}_{\text{matter}} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \partial_\mu \eta \partial^\mu \eta + \mathcal{L}_Y - V$$

$$\mathcal{L}_Y = y_\phi \phi \bar{N}^c N + y_\eta \eta \bar{N}^c N$$

$$V = \frac{\lambda_\phi}{4} \phi^4 + \frac{\lambda_{\phi\eta}}{4} \eta^2 \phi^2 + \frac{\lambda_\eta}{4} \eta^4$$

- ▶ Full classical scale invariance  $\rightarrow M_P$  dynamically
- ▶ The running of  $\lambda_\phi$  allows  $v_\phi \neq 0 \Rightarrow \phi = v_\phi + \varphi$

$$\Rightarrow f(\phi) \rightarrow f(\varphi + v_\phi) = \xi_\phi (\varphi + v_\phi)^2 / 2 \quad \Rightarrow \quad \boxed{M_P^2 = \xi_\phi v_\phi^2}$$

- ▶ inflaton:  $\phi$
- ▶  $N$  not needed for CW inflation itself, but for our particular case

# From the Jordan frame to the Einstein frame

- Conformal transformation:

$$g_{\mu\nu} \rightarrow \Omega(\phi)^2 g_{\mu\nu}$$
$$\Omega(\phi)^2 = \frac{2}{M_P^2} f(\phi)$$

- The scalar potential in the Einstein frame is given by

$$U = \frac{V(\phi)}{\Omega(\phi)^4} = \frac{\lambda_\phi(\phi)\phi^4}{4\Omega(\phi)^4}$$

- Canonically normalised field  $\chi$

$$\frac{d\chi}{d\phi} = M_P \sqrt{\frac{f(\phi) + 3f(\phi)^2}{2f(\phi)^2}}$$



# Scalar potential $U(\phi)$

- ▶ Since we live in the minimum with non-zero Planck scale:  $\phi = \varphi + v_\phi$ . Moreover during inflation  $\eta = 0 \rightarrow$  back to this point later.

$$\left. \begin{aligned} V(\varphi) &= \lambda_\phi (\varphi + v_\phi)^4 \\ f(\varphi) &= \frac{1}{2}(\varphi + v_\phi)^2 \end{aligned} \right\} \Rightarrow \boxed{U = \frac{1}{4} \lambda_\phi \frac{M_P^4}{\xi_\phi^2}}$$

- ▶ Slow-roll parameters

$$\epsilon = \frac{M_P^2}{2} \left( \frac{U'}{U} \frac{1}{d\chi/d\varphi} \right)^2$$

$$\eta = \frac{M_P^2}{U} \frac{d^2 U}{d\chi^2} = \frac{M_P^2}{U} \left[ U'' \left( \frac{d\chi}{d\varphi} \right) - U' \left( \frac{d\chi}{d\varphi} \right)' \right] / \left( \frac{d\chi}{d\varphi} \right)^3$$

- ▶  $N = \frac{1}{\sqrt{2}M_P} \int_{\varphi_{\text{end}}}^{\varphi^*} \frac{d\varphi}{\sqrt{\epsilon}}$  where  $d\varphi = \frac{d\chi}{d\chi/d\varphi}$

- ▶ now we can apply the recipe given before

# Multiple Point Criticality Principle

A generic full classical scale invariant theory implies:

- ▶  $V \sim \lambda_{ijkl} \phi_i \phi_j \phi_k \phi_l$
- ▶  $\Lambda = 0$

Therefore problems at the minimum:

- $a. v_i = 0 \Rightarrow \begin{cases} V(v_i) = 0 \rightarrow \text{OK: avoid eternal inflation} \\ M_P = 0 \rightarrow \text{unphysical!} \end{cases}$
- $b. v_i \neq 0 \Rightarrow \begin{cases} V(v_i) \neq 0 \rightarrow \text{bad: eternal inflation} \\ M_P \neq 0 \rightarrow \text{physical} \end{cases}$

Unless we also impose MPCP so that in addition to the trivial solution  $v_i = 0$  we also get

$$v_i \neq 0 \Rightarrow \begin{cases} V(v_i) = 0 \rightarrow \text{OK: avoid eternal inflation} \\ M_P \neq 0 \rightarrow \text{physical!} \end{cases}$$

# Multiple Point Criticality Principle

- ▶ Old concept. Used for predicting the Higgs mass from boundary conditions at the Planck scale:  $\lambda_H(M_P) = \beta_{\lambda_H}(M_P) = 0$   
C. D. Froggatt, H. B. Nielsen, Phys.Lett. B368 (1996)
- ▶ Same idea already applied for Higgs inflation: Haba et al. 1406.0158  
(tuning of top mass, scalar singlet and right handed neutrino couplings)
- ▶ MPCP is general feature for all full classical scale invariant models of inflation.

# MPCP and $U(\phi)$

- ▶ full classical scale invariance  $\rightarrow U(v_\phi) \sim 0$
- ▶ induce the minimum in  $U$  via a minimum in  $\lambda_\phi$ .

$$U = \frac{1}{4} \lambda_\phi \frac{M_P^4}{\xi_\phi^2}$$

Therefore we need to impose:

- $\lambda_\phi(v_\phi) = 0 \rightarrow$  ensure a small vacuum energy after inflation  
(condition on the RGE of  $\lambda_\phi$ )
- $\lambda'_\phi(v_\phi) = 0 \rightarrow$  minimum of  $\lambda \Rightarrow$  minimum of  $U$   
(condition on the RGE of  $\lambda_{\phi\eta}$  and  $y_\phi$ :  $N$  contribution crucial!)

# RGEs

From R. N. Lerner, J. McDonald, Phys.Rev.D80:123507, 2009

It can be shown that the commutation relation for an arbitrary scalar  $\phi$

$$\pi = \frac{\partial L}{\partial \dot{\phi}} = \sqrt{-\tilde{g}} \left( \frac{d\chi}{d\phi} \right)^2 \eta_{\mu} \tilde{g}^{\mu\nu} \tilde{\partial}_{\nu} \phi$$

$$[\phi(\vec{x}), \pi(\vec{y})] \equiv \Omega^2 \left( \frac{d\chi}{d\phi} \right)^2 \sqrt{-g} [\phi, \dot{\phi}] = i \hbar \delta^3(\vec{x} - \vec{y})$$

This implies that the scalar propagator will be suppressed by a factor

$c(\phi) = \frac{1}{\Omega^2 \left( \frac{d\chi}{d\phi} \right)^2}$ . In our case

$$c_{\varphi} = \frac{1 + \frac{\xi_{\phi} \varphi^2}{M_p^2}}{1 + (6\xi_{\phi} + 1) \frac{\xi_{\phi} \varphi^2}{M_p^2}}$$

When calculating the RG equations or Coleman-Weinberg potential, one suppression factor is inserted for each  $\varphi$  propagator in a loop.

## RGEs II

At one-loop level, the  $\beta$ -functions for the scalar coupling and the non-minimal coupling of  $\phi$  are given by

$$16\pi^2\beta_{\lambda_\phi} = 18c_\varphi^2\lambda_\phi^2 + \frac{1}{2}\lambda_{\phi\eta}^2 + 16c_\varphi\lambda_\phi y_\phi^2 - 64y_\phi^4$$

$$16\pi^2\beta_{\lambda_\eta} = 18\lambda_\eta^2 + \frac{1}{2}c_\varphi^2\lambda_{\phi\eta}^2 + 16\lambda_\eta y_\eta^2 - 64y_\eta^4$$

$$16\pi^2\beta_{\lambda_{\phi\eta}} = 4c_\varphi\lambda_{\phi\eta}^2 + 6\lambda_{\phi\eta}(c_\varphi^2\lambda_\phi + \lambda_\eta) + 8\lambda_{\phi\eta}(c_\varphi y_\phi^2 + y_\eta^2) - 384y_\phi^2 y_\eta^2$$

$$16\pi^2\beta_{y_\phi} = 16y_\phi(c_\varphi y_\phi^2 + y_\eta^2),$$

$$16\pi^2\beta_{y_\eta} = 16y_\eta(c_\varphi y_\phi^2 + y_\eta^2)$$

$$16\pi^2\beta_{\xi_\phi} = 6c_\varphi\left(\xi_\phi + \frac{1}{6}\right)\lambda_\phi$$

For numerical purposes

- ▶ neglect  $\lambda_\phi$  in the RGEs
- ▶  $\xi_\phi \in [0, 0.0115] \rightarrow c_\varphi \simeq 1$

## RGEs II

At one-loop level, the  $\beta$ -functions for the scalar coupling and the non-minimal coupling of  $\phi$  are given by

$$16\pi^2\beta_{\lambda_\phi} = \frac{1}{2}\lambda_{\phi\eta}^2 - 64y_\phi^4$$

$$16\pi^2\beta_{\lambda_\eta} = 18\lambda_\eta^2 + \frac{1}{2}\lambda_{\phi\eta}^2 + 16\lambda_\eta y_\eta^2 - 64y_\eta^4$$

$$16\pi^2\beta_{\lambda_{\phi\eta}} = 4\lambda_{\phi\eta}^2 + 6\lambda_{\phi\eta}\lambda_\eta + 8\lambda_{\phi\eta}(y_\phi^2 + y_\eta^2) - 384y_\phi^2y_\eta^2$$

$$16\pi^2\beta_{y_\phi} = 16y_\phi(y_\phi^2 + y_\eta^2),$$

$$16\pi^2\beta_{y_\eta} = 16y_\eta(y_\phi^2 + y_\eta^2)$$

$$16\pi^2\beta_{\xi_\phi} = 0$$

For numerical purposes

- ▶ neglect  $\lambda_\phi$  in the RGEs  $\Rightarrow \xi_\phi \sim \text{constant}$
- ▶  $\xi_\phi \in [0, 0.0115] \rightarrow c_\phi \simeq 1$

# Minimum condition

- ▶ The  $\beta$ -functions are logarithmic derivatives of couplings:  $\beta_{\lambda_i} = \mu \frac{d\lambda_i}{d\mu}$ .
- ▶ Using  $\mu = \phi$ :

$$\lambda'_{\phi}(v_{\phi}) = \frac{\beta_{\lambda_{\phi}}(v_{\phi})}{\phi} = 0 \quad \Rightarrow \quad \beta_{\lambda_{\phi}}(v_{\phi}) = 0 \quad \Rightarrow \quad \boxed{\frac{1}{2}\lambda_{\phi\eta}^2 - 64y_{\phi}^4 \simeq 0}$$

- ▶ However this only ensures that  $\phi = v_{\phi}$  is a stationary point. To make it a minimum, we need to impose  $\lambda''_{\phi}(v_{\phi}) > 0$

$$\lambda''_{\phi} = \frac{d}{d\mu} \left( \frac{\beta_{\lambda_{\phi}}}{\mu} \right) = \frac{1}{\mu} \frac{d\beta_{\lambda_{\phi}}}{d\mu} - \frac{\beta_{\lambda_{\phi}}}{\mu^2}$$

$$\lambda''_{\phi}(v_{\phi}) = \frac{\beta'_{\lambda_{\phi}}(v_{\phi})}{v_{\phi}} \Rightarrow \dots$$

$$\boxed{\xi_{\phi} \lambda_{\phi\eta} \left[ 12\lambda_{\eta} + (8 - 3\sqrt{2})\lambda_{\phi\eta} - 48(1 + \sqrt{2})y_{\eta}^2 \right] > 0}$$



# More on the parameters

- ▶  $\xi_\phi \in [0, 0.0115] \Rightarrow v_\phi > M_P$   
 $\Rightarrow$  we need to assume that quantum gravity is weakly coupled and subdominant
- ▶  $0 \leq \lambda_\eta < \frac{2}{3}\pi$  for perturbativity and in order to avoid a VEV for  $\eta$  (DM?).  
 $\lambda_\eta$  fixed by the constraint on  $U(\varphi^*)$ .
- ▶  $y_\eta = 0$ . We wanted  $\eta$  to be a superheavy DM candidate (WIMPZILLA).  
Unluckily it is too heavy even for a WIMPZILLA. Negligible relic density.  
(the case  $y_\eta \neq 0$  is under study)
- ▶  $\lambda_{\phi\eta}$ ,  $y_\phi$  are fixed via the boundary condition  $\lambda_\phi(v_\phi) = \beta_{\lambda_\phi}(v_\phi) = 0$
- ▶  $\eta = 0$  during inflation

## More on $U(\phi)$

► Einstein frame potential:  $U = \frac{1}{4} \lambda_\phi \frac{M_{\text{P}}^4}{\xi_\phi^2}$

►  $m_\phi^2 = U''(v_\phi) = \frac{1}{8} \frac{\lambda_\phi''(v_\phi) M_{\text{P}}^4}{\xi_\phi^2}$

►  $m_\eta^2 = \frac{1}{2} \lambda_{\phi\eta} v_\phi^2 = \frac{1}{2} \lambda_{\phi\eta} \frac{M_{\text{P}}^2}{\xi_\phi}$

► The slow-roll parameters

$$\epsilon = \frac{(\lambda'_\phi)^2}{\lambda_\phi^2} \frac{(M_{\text{P}} + \sqrt{\xi_\phi} \varphi)^2}{2(1 + 6\xi_\phi) \varphi^2}$$

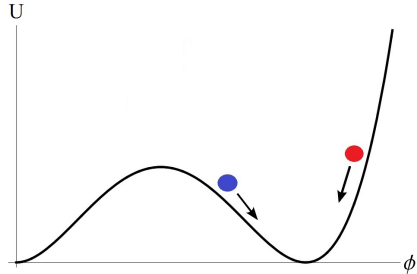
$$\eta = \frac{\xi_\phi (v_\phi + \varphi) [\lambda'_\phi + (v_\phi + \varphi) \lambda''_\phi]}{\lambda_\phi (1 + 6\xi_\phi)}$$

# Shape of potential

$$S = \int d^4x \sqrt{-g} [f(\phi)R + \mathcal{L}_{\text{matter}}]$$

$$f(\phi) = \frac{\xi\phi}{2}\phi^2$$

$$\mathcal{L}_{\text{matter}} = \text{same as before}$$



► Such a shape allows for two different, generic types of inflation:

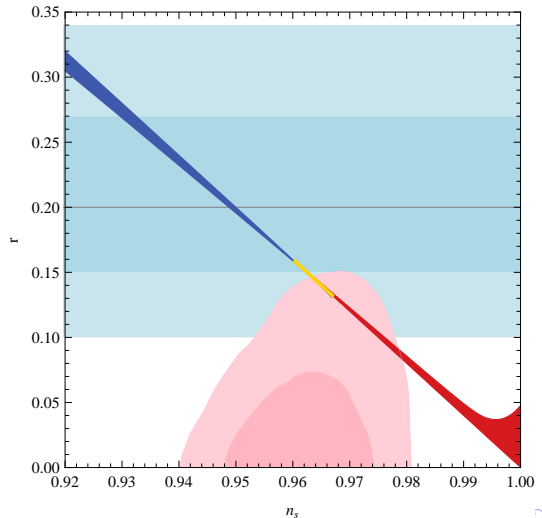
- i. Small-field inflation, when  $\phi$  rolls forward down to  $v_\phi$ : ●
- ii. Large-field (chaotic) inflation, when  $\phi$  rolls back down to  $v_\phi$ : ●

# Results

$$N \in [50, 60]$$

$$\xi_\phi \in [0, 0.0115]$$

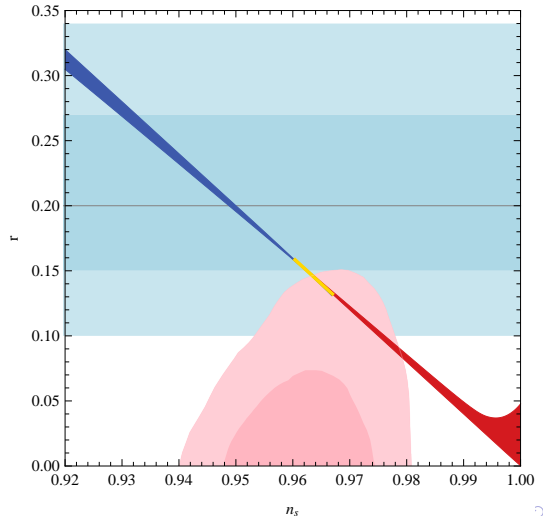
- Planck bound
- BICEP2 bound
- $V = m^2 \phi^2$
- large field
- small field



# Other results

In the allowed region:

- ▶ large field case favoured
- ▶  $m_\phi \sim 10^{13}$  GeV
- ▶  $\lambda_{\phi\eta} \sim 10^{-4}$
- ▶  $\xi_\phi \sim 10^{-3}$
- ▶  $m_\eta \sim M_P$   
 $\rightarrow \eta = 0$  during inflation
- ▶  $T_{RH} \sim 10^{12,13}$  GeV

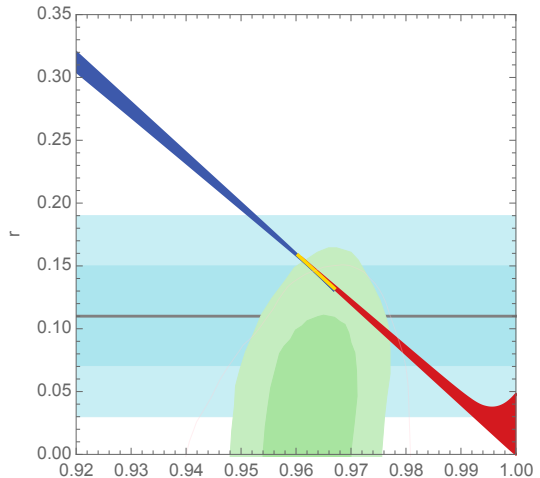


# Results and new constraints

$$N \in [50, 60]$$

$$\xi_\phi \in [0, 0.0115]$$

- 1409.7870
- 1409.4491
- $V = m^2 \phi^2$
- large field
- small field



# Conclusions

- ▶ We constructed a full classical scale invariant model
- ▶ MPCP is the guideline to combine full classical scale invariance and inflation
- ▶ Found region in agreement with BICEP2 & Planck
- ▶ large field case favoured
- ▶ Found region in agreement with the new data analysis
- ▶ Predictive model that can be confirmed or ruled out

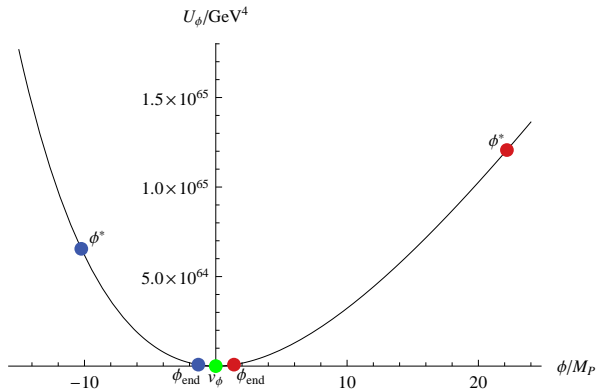
Thank you!





# Backup slides

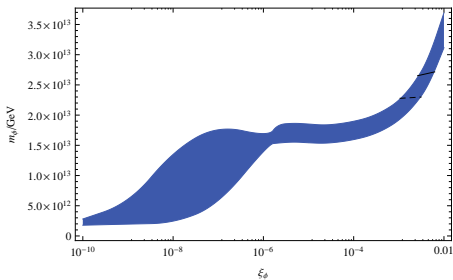
# $U_\phi$ around $v_\phi$



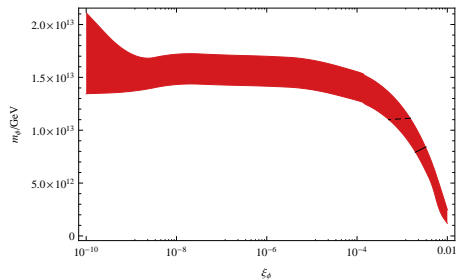
- Around the VEV, we can expand:

$$\lambda_\phi(\phi) = \lambda_\phi(v_\phi) + \lambda'_\phi(v_\phi)(\phi - v_\phi) + \frac{1}{2}\lambda''_\phi(v_\phi)(\phi - v_\phi)^2 + \mathcal{O}(\phi^3)$$

# $m_\phi$ vs $\xi_\phi$

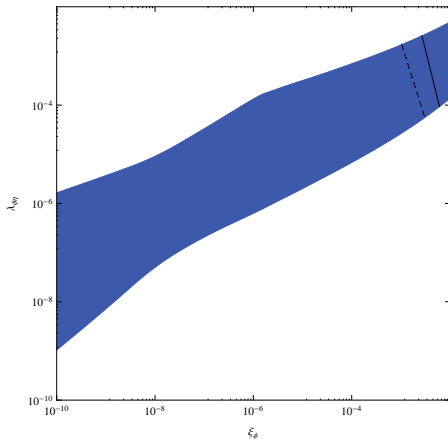


■ small field

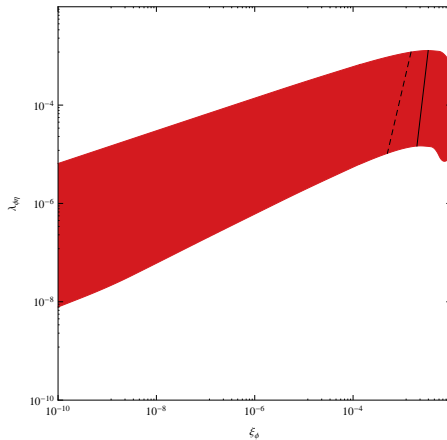


■ large field

# $\lambda_{\phi\eta}$ vs $\xi_\phi$

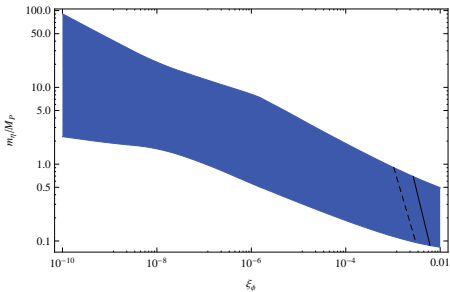


■ small field

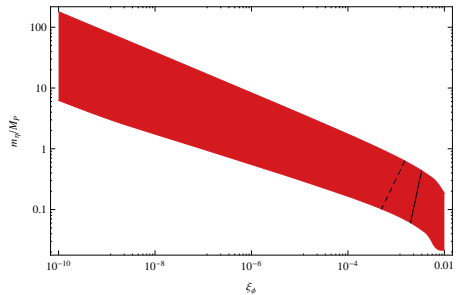


■ large field

# $m_\eta$ vs $\xi_\phi$

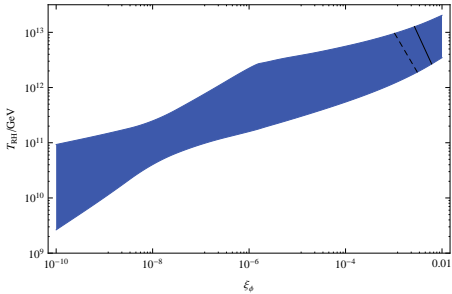


■ small field

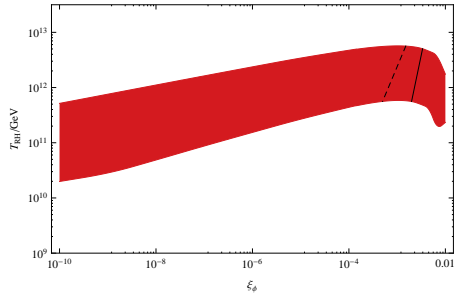


■ large field

# $T_{RH}$ vs $\xi_\phi$



■ small field



■ large field