Inflation and CMB polarizations

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Summary

Cosmological Observables

All current cosmological data sets are consistent with a 6-parameter model (ΛCDM):

- $\{\Omega_b, \Omega_{\rm CDM}, h, \tau\}$ describe the homogeneous background
- $\{A_s, n_s\}$ characterize the primordial density fluctuations

Planck 2015 Data

Label	Definition	Physical Origin	Value
$\Omega_b h^2$	Baryon Fraction	Baryogenesis	0.02222 ± 0.00023
$\Omega_{\rm CDM}$	DM Fraction	TeV-Scale Physics (?)	0.1197 ± 0.0022
τ	Optical Depth	First Stars	0.078 ± 0.019
h	Hubble Parameter	Cosmological Epoch	0.6731 ± 0.0096
$\ln(10^{10} A_s)$	Scalar Amplitude	Inflation	(3.089 ± 0.036)
n_s	Scalar Index	Inflation	0.9655 ± 0.0062

- flat universe: $\Omega_b + \Omega_{\rm CDM} + \Omega_\Lambda \equiv 1$
- Ω_Λ Dark Energy fraction
- $A_s \simeq 2.14 \times 10^{-9}$

Cosmological Observables

The minimal set of parameters is not fixed, but is dictated by the quality of the available data and our knowledge/ignorance of fundamental physical parameters and interactions. Future possible parameters are:

Label	Definition	Physical Origin
Ω_k	Curvature	Initial Conditions
Σm_ν	Neutrino Mass	Beyond-SM Physics
w	Dark Energy Equation of State	Unknown
N_ν	Neutrino-like Species	Beyond-SM Physics
Y_{He}	Helium Fraction	Nucleosynthesis
α_s	Scalar "Running"	Inflation
A_t	Tensor Amplitude	Inflation
n_t	Tensor Index	Inflation
f_{NL}	Non-Gaussianity	Inflation (?)
S	Isocurvature	Inflation
$G\mu$	Topological Defects	Phase Transition

Why inflation?

It solves

- *Flatness Problem*: Present observations show that the universe is very nearly spatially flat. To explain the geometric flatness of space today therefore requires an extreme fine-tuning in a Big Bang cosmology without inflation.
- *Horizon Problem*: Observations of the CMB imply the existence of temperature correlations across distances on the sky that corresponded to super-horizon scales at the time when the CMB radiation was released. Yet there is no way to establish thermal equilibrium if these points were never in causal contact before last scattering.
- . . .

(cft Kristjan's seminar)

Physics of inflation

The Friedmann equations governing the scale factor $a(t)$ of a spatially flat universe with Friedmann-Robertson-Walker (FRW) metric

$$\delta s^2 = -\delta t^2 + a(t)^2 \delta \mathbf{x}^2$$

are

$$\begin{aligned} H^2 &= \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_P^2}\rho \\ \dot{H} + H^2 &= \frac{\ddot{a}}{a} = -\frac{1}{6M_P^2}(\rho + 3p) \end{aligned}$$

What drives the accelerated expansion of the early universe?

Inflation requires a source of negative pressure p and an energy density ρ which dilutes very slowly allowing for an exit into the standard Big Bang cosmology at later times.

Single-field slow-roll inflation

- $\mathcal{L}_{\rm eff}(\phi) = \frac{1}{2}(\partial\phi)^2 - V(\phi) \xrightarrow[\textit{isotropic}]{\text{homogeneous}} \mathcal{L}_{\rm eff}(\phi) = \frac{1}{2}\dot{\phi}^2 - V(\phi)$
- EoM:

$$\begin{aligned} H^2 &= \frac{1}{3M_P^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) \\ \frac{\ddot{a}}{a} &= -\frac{1}{3M_P^2}\left(\dot{\phi}^2 - V(\phi)\right) \end{aligned}$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

- $\ddot{a} > 0 \;\Leftrightarrow\; V \gg \dot{\phi}^2 \qquad \Leftarrow |\ddot{\phi}| \ll |V'|$.
- Quantitatively, inflation requires smallness of the slow-roll parameters:

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{M_P^2}{2}\frac{\dot{\phi}^2}{H^2} \approx \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1 \qquad |\eta| \approx M_{\rm pl}^2\left|\frac{V''}{V}\right| \ll 1$$

- Approx. solution:

$$a(t) \approx a(0)e^{Ht}, \qquad H \approx \mathrm{const} \qquad \Rightarrow \text{Exponential expansion!}$$

Single-field slow-roll inflation

- Once these constraints are satisfied, the inflationary process (and its termination) happens generically for a wide class of models:

$$a(t) \approx a(0)e^{Ht}\,, \qquad H \approx \text{const}$$

- For inflation to successfully address the Big Bang problems, one must simply ensure that the inflationary process produces a sufficient number of these '*e*-folds' of accelerated expansion

$$N_e \equiv \ln\left(\frac{a(t_{\text{final}})}{a(t_{\text{initial}})}\right)$$

- A typical range for the required number of *e*-folds is $N_e \in [50, 60]$
- for more details see Kristjan's seminar

SVT Decomposition

We define perturbations around the homogeneous solutions for $\bar{\phi}(t)$ and $\bar{g}_{\mu\nu}(t)$,

$$\phi(t,\mathbf{x}) = \bar{\phi}(t) + \delta\phi(t,\mathbf{x})\,, \qquad g_{\mu\nu}(t,\mathbf{x}) = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(t,\mathbf{x})$$

where

$$\begin{aligned} \delta s^2 &= g_{\mu\nu}\,\delta x^\mu \delta x^\nu \\ &= -(1+2\Phi)\delta t^2 + 2aB_i\delta x^i\delta t + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]\delta x^i\delta x^j \end{aligned}$$

We can decompose the perturbations into independent scalar (S), vector (V) and tensor (T) modes. This SVT decomposition is most easily described in Fourier space

$$Q_{\mathbf{k}}(t) = \int \delta^3\mathbf{x}\;Q(t,\mathbf{x})\,e^{i\mathbf{k}\cdot\mathbf{x}}\,, \qquad Q \equiv \delta\phi,\ \delta g_{\mu\nu}$$

SVT Decomposition

$$\boxed{\delta s^2 = -(1+2\Phi)\delta t^2 + 2aB_i\delta x^i\delta t + a^2[(1-2\Psi)\delta_{ij} + E_{ij}]\delta x^i\delta x^j}$$

In real space, the SVT decomposition of the metric perturbations

$$\begin{aligned} B_i &\equiv \partial_i B - S_i\,, \qquad \text{where} \quad \partial^i S_i = 0 \\ E_{ij} &\equiv 2\partial_{ij}E + 2\partial_{(i}F_{j)} + h_{ij}\,, \qquad \text{where} \quad \partial^i F_i = 0\,, \quad h^i_i = \partial^i h_{ij} = 0 \end{aligned}$$

- perturbations of each type evolve independently (at the linear level)
- $\delta\phi$ and $\delta g_{\mu\nu}$ are gauge(frame)-dependent. Physical questions therefore have to be studied in a fixed gauge or in terms of gauge-invariant quantities. An important gauge-invariant quantity is the curvature perturbation on uniform-density hypersurfaces

$$-\zeta \equiv \Psi + \frac{H}{\dot\rho}\delta\rho$$

where ρ is the total energy density of the universe.

S(calar) perturbations

In a gauge where $\delta\rho_\phi = 0$ all scalar degrees of freedom can be expressed by

$$g_{ij} = a^2(t)[1 + 2\zeta]\delta_{ij}$$

(the fluctuations in $g_{\mu 0}$ are related to ζ by Einstein's equations)
A crucial statistical measure of ζ is the power spectrum of ζ

$$\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \rangle = (2\pi)^3 \, \delta(\mathbf{k} + \mathbf{k}') \, \frac{2\pi^2}{k^3} P_s(k)$$

The scale-dependence of the power spectrum is defined by

$$n_s - 1 \equiv \frac{d \ln P_s}{d \ln k} \qquad (n_s = 1 \Leftrightarrow \text{ scale invariance})$$

The power spectrum is often approximated by a power law form

$$P_s(k) = A_s(k_\star) \left(\frac{k}{k_\star} \right)^{n_s(k_\star) - 1 + \frac{1}{2}\alpha_s(k_\star) \ln(k/k_\star)}$$

where $k_\star$ is the pivot scale and $\alpha_s \equiv dn_s/d \ln k$ the running of n_s.

S(calar) perturbations

- If ζ is Gaussian then $P_s(k)$ contains all the statistical information.
- Primordial non-Gaussianity is encoded in $\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \zeta_{\mathbf{k}''} \ldots \rangle$.
- In single-field slow-roll inflation the non-Gaussianity is predicted to be small
- non-Gaussianity can be significant in multi-field models or in single-field models with non-trivial kinetic terms and/or violation of the slow-roll conditions.

V(ector) perturbations

The vector perturbations S_i and F_i are distinguished from the scalar perturbations B, Ψ and E as they are divergence-free, *i.e.* $\partial^i S_i = \partial^i F_i = 0$.

One may show that vector perturbations on large scales are redshifted away by Hubble expansion (unless they are driven by anisotropic stress). In particular, vector perturbations are subdominant at the time of recombination.

Since CMB polarization is generated at last scattering the polarization signal is dominated by scalar and tensor perturbations.

T(ensor) perturbations

They are uniquely described by a gauge-invariant metric perturbation h_{ij}

$$g_{ij} = a^2(t)[\delta_{ij} + h_{ij}]\,, \qquad \partial_j h_{ij} = h^i_i = 0$$

Physically, h_{ij} corresponds to gravitational wave fluctuations.
The power spectrum for the two polarization modes of $h_{ij} \equiv h^+ e^+_{ij} + h^\times e^\times_{ij}$, $h \equiv h^+, h^\times$, is defined as

$$\langle h_{\mathbf{k}} h_{\mathbf{k}'} \rangle = (2\pi)^3\, \delta(\mathbf{k} + \mathbf{k}')\, \frac{2\pi^2}{k^3} P_t(k)$$

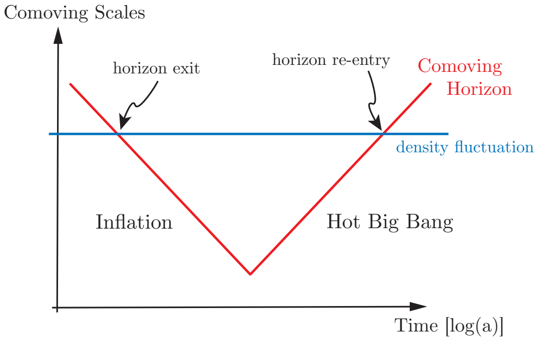
and its scale-dependence is defined as

$$n_t \equiv \frac{d \ln P_t}{d \ln k} \qquad\qquad i.e. \quad P_t(k) = A_t(k_\star) \left(\frac{k}{k_\star} \right)^{n_t(k_\star)}$$

CMB polarization measurements are sensitive to the ratio

$$r \equiv \frac{P_t}{P_s}$$

Quantum fluctuations



Quantum fluctuations during inflation are the source of $P_s(k)$ and $P_t(k)$. They are created on sub-horizon scales. While comoving scales, k^{-1}, remain constant the comoving Hubble radius during inflation, $(aH)^{-1}$, shrinks and the perturbations exit the horizon. Causal physics cannot act on superhorizon perturbations and they freeze until horizon re-entry at late times.

Quantum fluctuations

In spatially-flat gauge, perturbations in ζ are related to perturbations in $\delta\phi$

$$\zeta = -H\frac{\delta\rho}{\dot{\rho}} \approx \xrightarrow{\text{slow roll}} \approx -H\frac{\delta\phi}{\dot{\phi}} \equiv -H\delta t$$

The power spectrum of ζ and $\delta\phi$ are therefore related

$$\langle\zeta_{\mathbf{k}}\zeta_{\mathbf{k}'}\rangle = \left(\frac{H}{\dot{\phi}}\right)^2 \langle\delta\phi_{\mathbf{k}}\,\delta\phi_{\mathbf{k}'}\rangle \approx \xrightarrow{\text{slow roll}} \approx (2\pi)^3 \left(\frac{H}{\dot{\phi}}\right)^2 \delta(\mathbf{k}+\mathbf{k}')\,\frac{2\pi^2}{k^3}\left(\frac{H}{2\pi}\right)^2$$

The r.h.s. is to be evaluated at horizon exit of a given perturbation $k = aH$. Therefore ϕ fluctuations produce the following power spectrum for ζ

$$P_s(k) = \left(\frac{H}{\dot{\phi}}\right)^2 \left(\frac{H}{2\pi}\right)^2\Bigg|_{k=aH}$$

In addition, quantum fluctuations during inflation excite tensor metric perturbations h_{ij}

$$P_t(k) = \frac{8}{M_P^2}\left(\frac{H}{2\pi}\right)^2\Bigg|_{k=aH}$$

Slow roll predictions

(single field) slow-roll approximation $\Rightarrow$ $P_s(k)$ and $P_t(k) \leftrightarrow V(\phi)$.

$$\boxed{P_s(k) = \frac{1}{24\pi^2 M_P^4} \frac{V}{\epsilon}\bigg|_{k=aH}}, \qquad \boxed{n_s - 1 = 2\eta - 6\epsilon}$$

$$P_t(k) = \frac{2}{3\pi^2} \frac{V}{M_P^4}\bigg|_{k=aH}, \qquad n_t = -2\epsilon\,, \qquad \boxed{r = 16\epsilon}$$

We also point out the existence of a slow-roll consistency relation

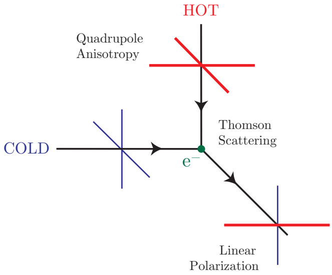
$$r = -8n_t$$

Measuring P_t ($\rightarrow V$), P_s ($\rightarrow V'$), n_s ($\rightarrow V''$) and α_s ($\rightarrow V'''$) allows a reconstruction of the inflaton potential as a Taylor expansion around $\phi_\star$ (corresponding to the time when fluctuations on CMB scales exited the horizon)

$$V(\phi) = V|_\star + V'|_\star(\phi - \phi_\star) + \frac{1}{2} V''|_\star(\phi - \phi_\star)^2 + \frac{1}{3!} V'''|_\star(\phi - \phi_\star)^3 + \cdots$$

where $(\dots)|_\star = (\dots)|_{\phi=\phi_\star}$.

Polarizations



Because the anisotropies in the CMB temperature are sourced by primordial fluctuations, we expect the CMB anisotropies to become polarized via Thomson scattering.

The polarization of CMB anisotropies is generated only by scattering therefore we can use polarization information to distinguish the different types of primordial perturbations.

If a free e^- 'sees' an incident radiation that is isotropic, then the outgoing radiation remains unpolarized because orthogonal polarization directions cancel out. However, if the incoming radiation field has a quadrupole component, a net linear polarization is generated via Thomson scattering. Since the temperature anisotropies are created by primordial fluctuations, a component of the polarization should be correlated with the temperature anisotropy.

Characterization of the radiation field

The anisotropy field is defined in terms of a 2×2 intensity tensor $I_{ij}(\hat{n})$, where $\hat{n}$ denotes the direction on the sky. The components of I_{ij} are defined relative to two orthogonal basis vectors $\hat{\mathbf{e}}_1$ and $\hat{\mathbf{e}}_2$ perpendicular to $\hat{n}$.

- Linear polarization is then described by the Stokes parameters $Q = \frac{1}{4}(I_{11} - I_{22})$ and $U = \frac{1}{2} I_{12}$
- the temperature anisotropy is $T = \frac{1}{4}(I_{11} + I_{22})$
- The polarization magnitude and angle are $P = \sqrt{Q^2 + U^2}$ and $\alpha = \frac{1}{2}\tan^{-1}(U/Q)$

T is invariant under a rotation in the plane perpendicular to $\hat{n}$ and hence may be expanded in terms of scalar (spin-0) spherical harmonics

$$T(\hat{n}) = \sum_{\ell,m} a^T_{\ell m} \, Y_{\ell m}(\hat{n}) \, .$$

Characterization of the radiation field

Q and U transform under rotation by an angle ψ as a spin-2 field $(Q \pm iU)(\hat{n}) \to e^{\mp 2i\psi}(Q \pm iU)(\hat{n})$. The harmonic analysis of $Q \pm iU$ therefore requires expansion on the sphere in terms of tensor (spin-2) spherical harmonics

$$(Q + iU)(\hat{n}) = \sum_{\ell,m} a_{\ell m}^{(\pm 2)} \left[{}_{\pm 2}Y_{\ell m}(\hat{n})\right] .$$

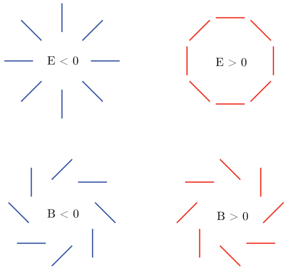
Instead of $a_{\ell m}^{(\pm 2)}$ it is convenient to introduce the linear combinations

$$a_{\ell m}^{E} \equiv -\frac{1}{2}\left(a_{\ell m}^{(2)} + a_{\ell m}^{(-2)}\right) , \qquad a_{\ell m}^{B} \equiv -\frac{1}{2i}\left(a_{\ell m}^{(2)} - a_{\ell m}^{(-2)}\right) .$$

Then one can define two scalar (spin-0) fields instead of the spin-2 quantities Q and U

$$E(\hat{n}) = \sum_{\ell,m} a_{\ell m}^{E}\ Y_{\ell m}(\hat{n}) , \qquad B(\hat{n}) = \sum_{\ell,m} a_{\ell m}^{B}\ Y_{\ell m}(\hat{n}) .$$

E- and B-modes



E-polarization is characterized as a *curl-free* mode with polarization vectors that are radial around cold spots and tangential around hot spots on the sky. B-polarization is *divergence-free* but has a *curl*: its polarization vectors have vorticity around any given point on the sky.
E and B are both invariant under rotations, but they behave differently under parity transformations.

E- and B-modes

The symmetries of temperature and polarization (E- and B-mode) anisotropies allow four types of correlations: the autocorrelations of temperature fluctuations and of E- and B-modes denoted by TT, EE, and BB, respectively, as well as the cross-correlation between temperature fluctuations and E-modes: TE. All other correlations (TB and EB) vanish for symmetry reasons.
The angular power spectra are defined as rotationally invariant quantities

$$C_\ell^{XY} \equiv \frac{1}{2\ell+1}\sum_m \langle a_{\ell m}^X a_{\ell m}^Y \rangle \,, \qquad X, Y = T, E, B$$

The dependence on cosmological parameters of each of these spectra differs, and hence a combined measurement of all of them greatly improves the constraints on cosmological parameters.

A smoking gun

It can be shown that:

i) scalar (density) perturbations create only E-modes and *no* B-modes.
ii) vector (vorticity) perturbations create mainly B-modes.
iii) tensor (gravitational wave) perturbations create both E-modes and B-modes.

The fact that scalars do not produce B-modes while tensors do is the basis for the often-quoted statement that detection of B-modes is a smoking gun of tensor modes, and therefore of inflation.

Power spectrum

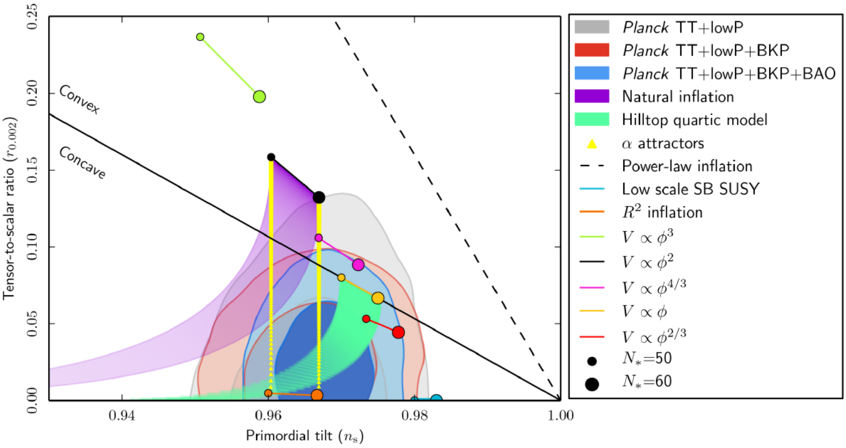
A power-law parameterization of the power spectrum is employed in

$$P_s(k) = A_s(k_\star)\left(\frac{k}{k_\star}\right)^{n_s(k_\star)-1+\frac{1}{2}\alpha_s(k_\star)\ln(k/k_\star)}$$

At $k_\star = 0.002\ \text{Mpc}^{-1}$, Planck 2015 Data says:

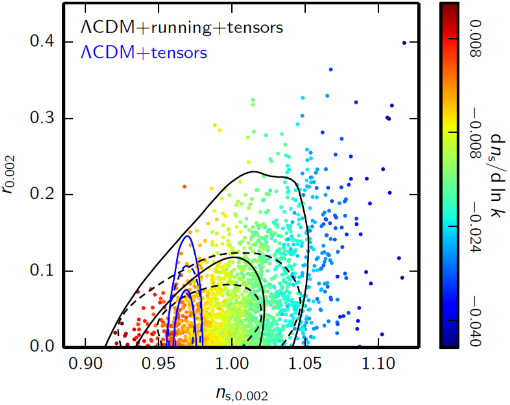
$$\begin{aligned} A_s &= (2.14 \pm 0.05) \times 10^{-9} \\ n_s &= 0.9655 \pm 0.0062 \\ \alpha_s &= \frac{dn_s}{d\ln k} = -0.008 \pm 0.016 \end{aligned}$$

r vs. n_s



lowP: $\ell < 30$, BAO: Baryon acoustic oscillations

r vs. n_s. $\alpha_S \neq 0$



* colored •: Planck TT+lowP
* black: Planck TT+lowP+lensing+BAO
* dashed: BICEP2/Keck/Planck
* blue: $\alpha_s = 0$

- Allowing for the running of n_S the allowed region increases considerably.

Conclusions

- The fact that scalars do not produce B-modes while tensors do is the basis for the often-quoted statement that detection of B-modes is a smoking gun of tensor modes, and therefore of inflation.
- The latest results for r vs. n_s suggest that we should give up the idea of a simple single field inflaton scenario and look for more complicated configurations: multifield inflation, $\alpha_S \neq 0$, ...

Thank you!

Backup slides