

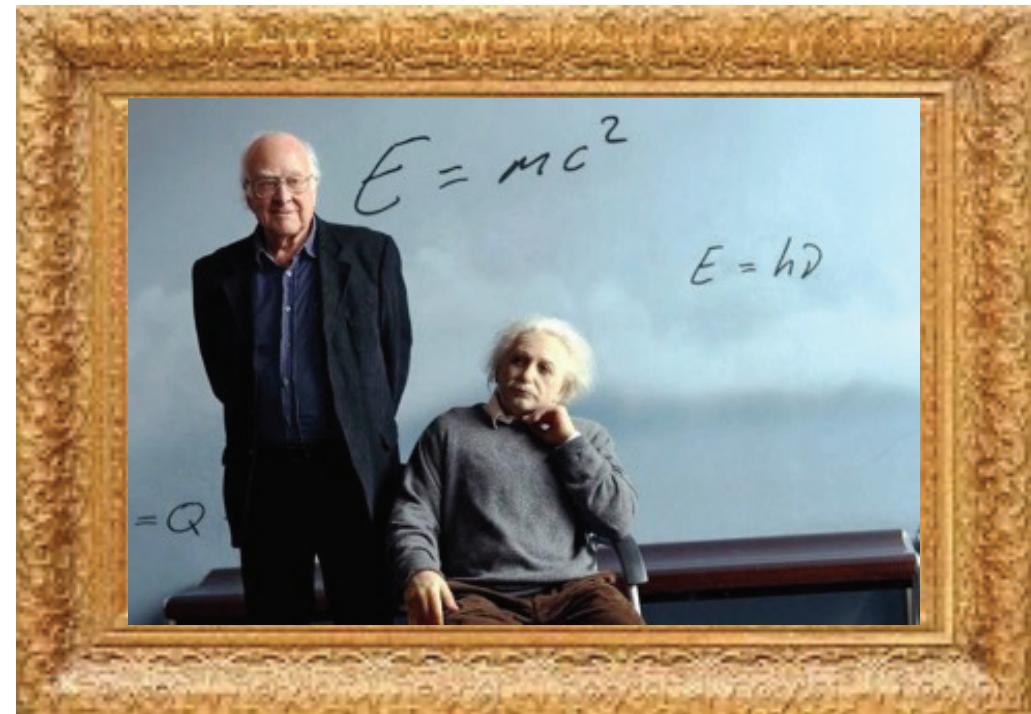
Framing Higgs inflation

w/ Damien George, Sander Mooij & Marco Volponi
1207.6963, 1310.2157, 1407.6874

Marieke Postma
Nikhef, Amsterdam



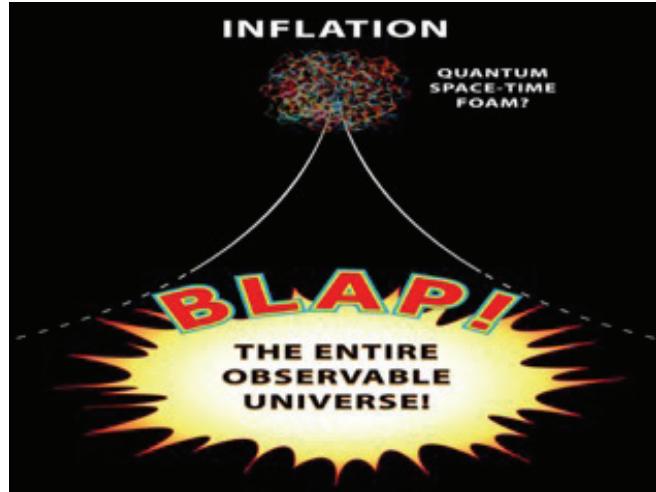
Tallinn
March 2015



Plan

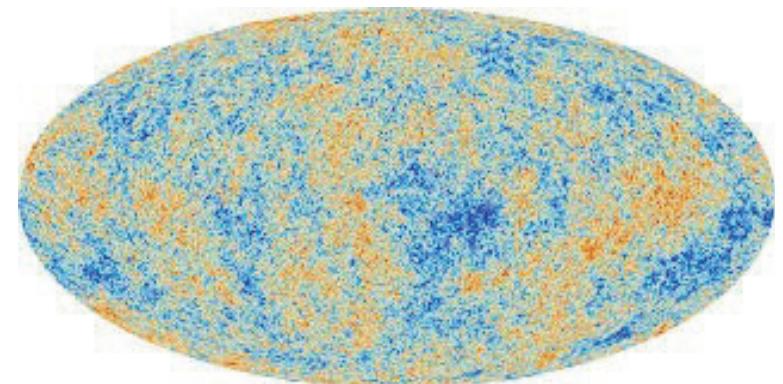
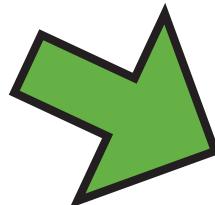
- Higgs inflation
- Frames

Inflation



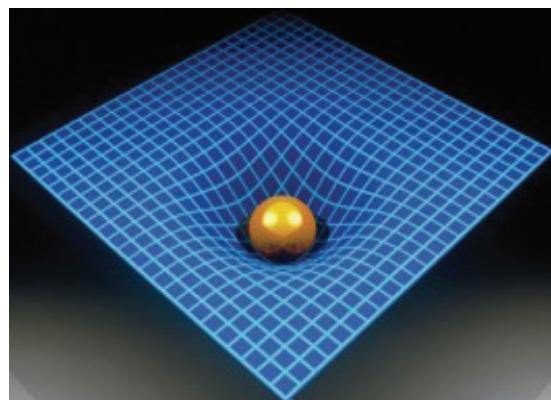
- homogeneous, flat universe
- small primordial fluctuations

$$\frac{\delta T}{T} \sim 10^{-5}$$



Inflation

$$a \sim e^{Ht}$$



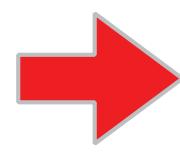
$$G_{\mu\nu}[g_{\mu\nu}] = 8\pi G_N T_{\mu\nu}$$



Gas Liquid Solid

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2$$

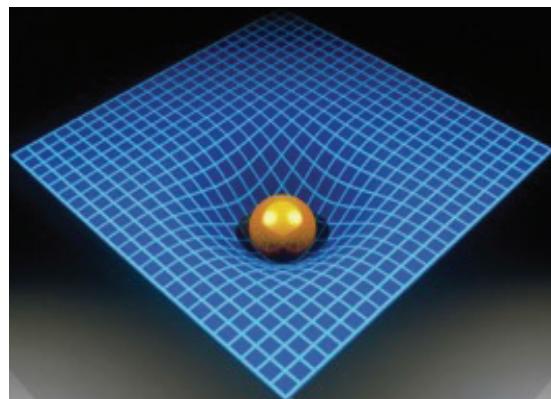
$$T_\mu^\nu = \text{diag}(\rho, -p, -p, -p)$$


$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{\rho}{3}$$

Inflation

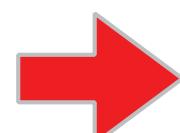
$$a \sim e^{Ht}$$

$$G_{\mu\nu}[g_{\mu\nu}] = 8\pi G_N T_{\mu\nu} \approx 8\pi G_N g_{\mu\nu} V(\phi)$$



$$ds^2 = dt^2 - a^2(t)d\vec{x}^2$$

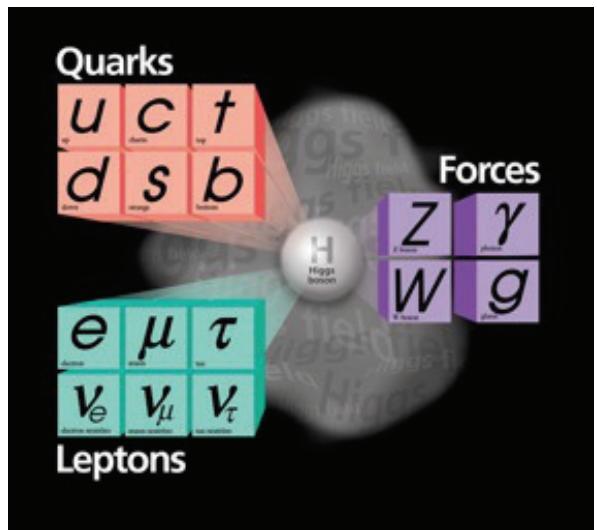
$$T_\mu^\nu = \text{diag}(\rho, -p, -p, -p)$$



$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{\rho}{3}$$

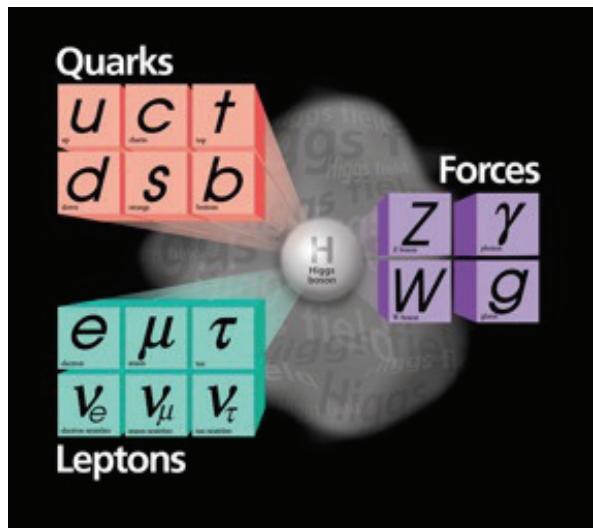
vacuum energy (scalar)

Higgs inflation?



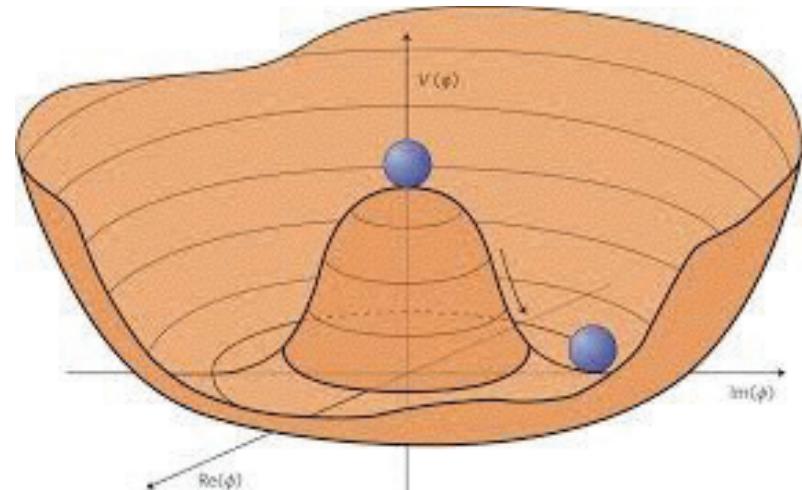
only scalar field

Higgs inflation?



only scalar field

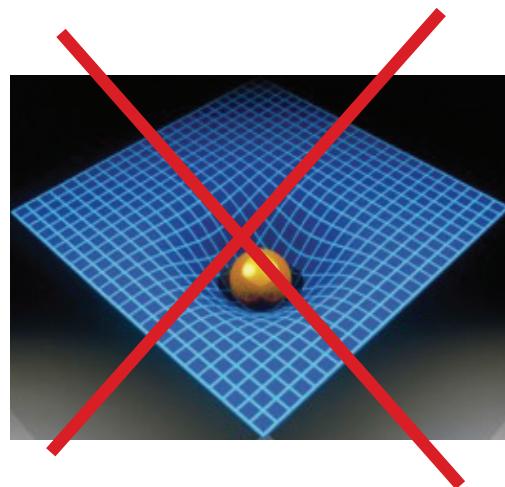
but



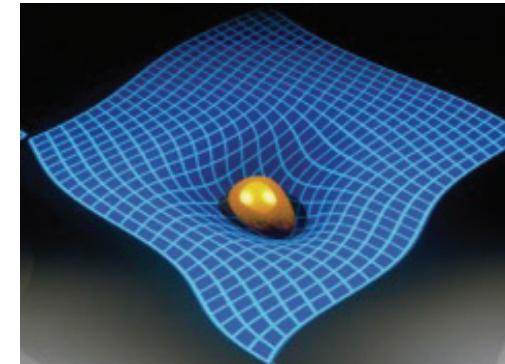
too steep!

Higgs inflation?

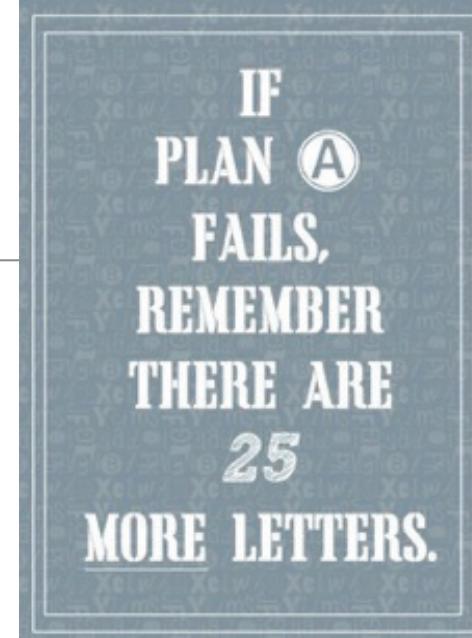
not



but



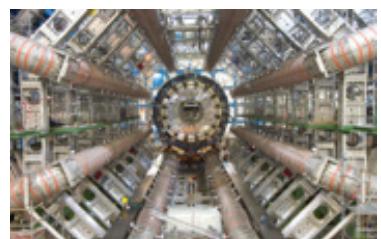
couple non-minimal to gravity



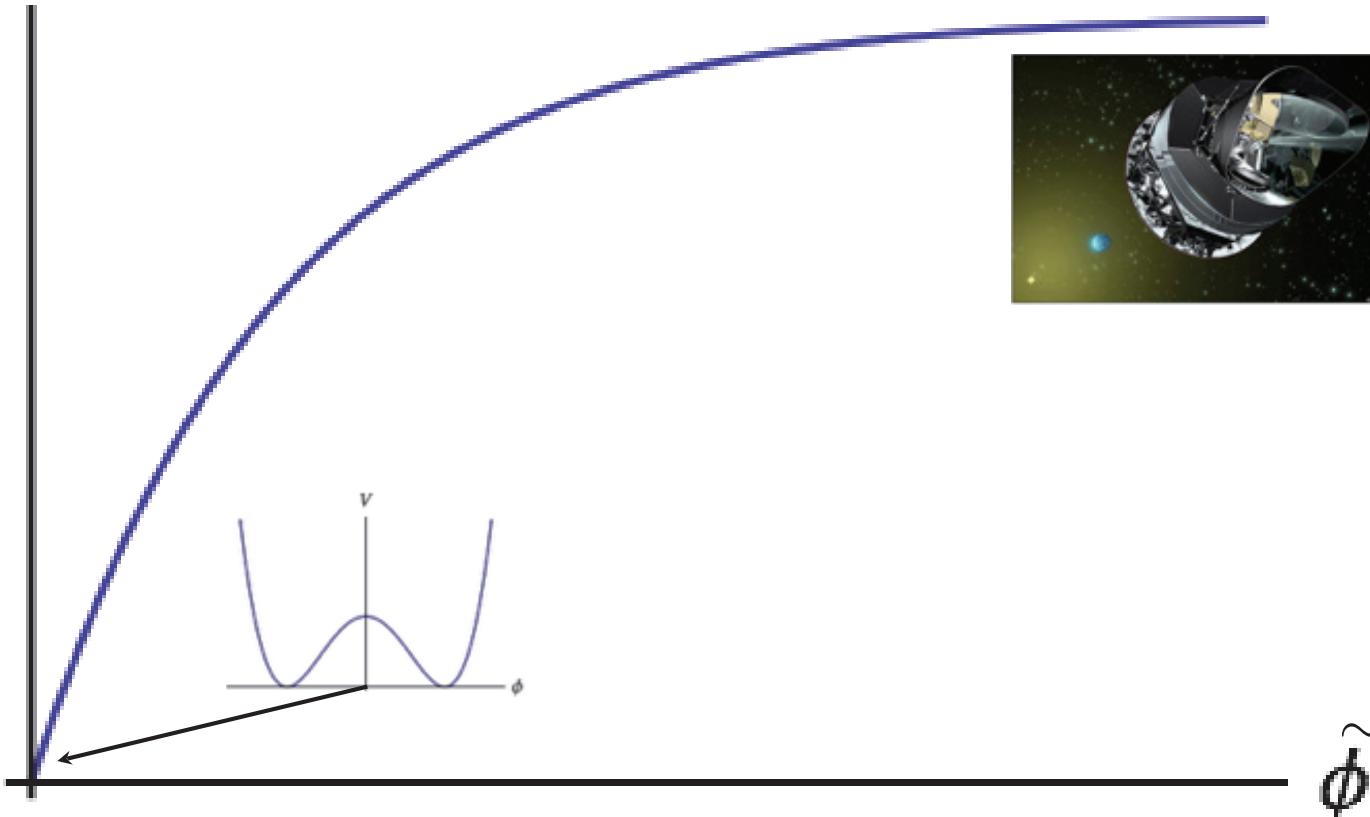
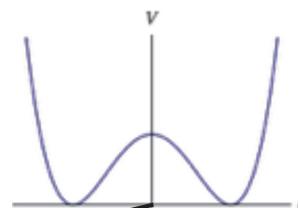
Higgs inflation

Fakir '83, Salopek, Bond, Bardeen '89, Bezrukov & Shaposhnikov '08

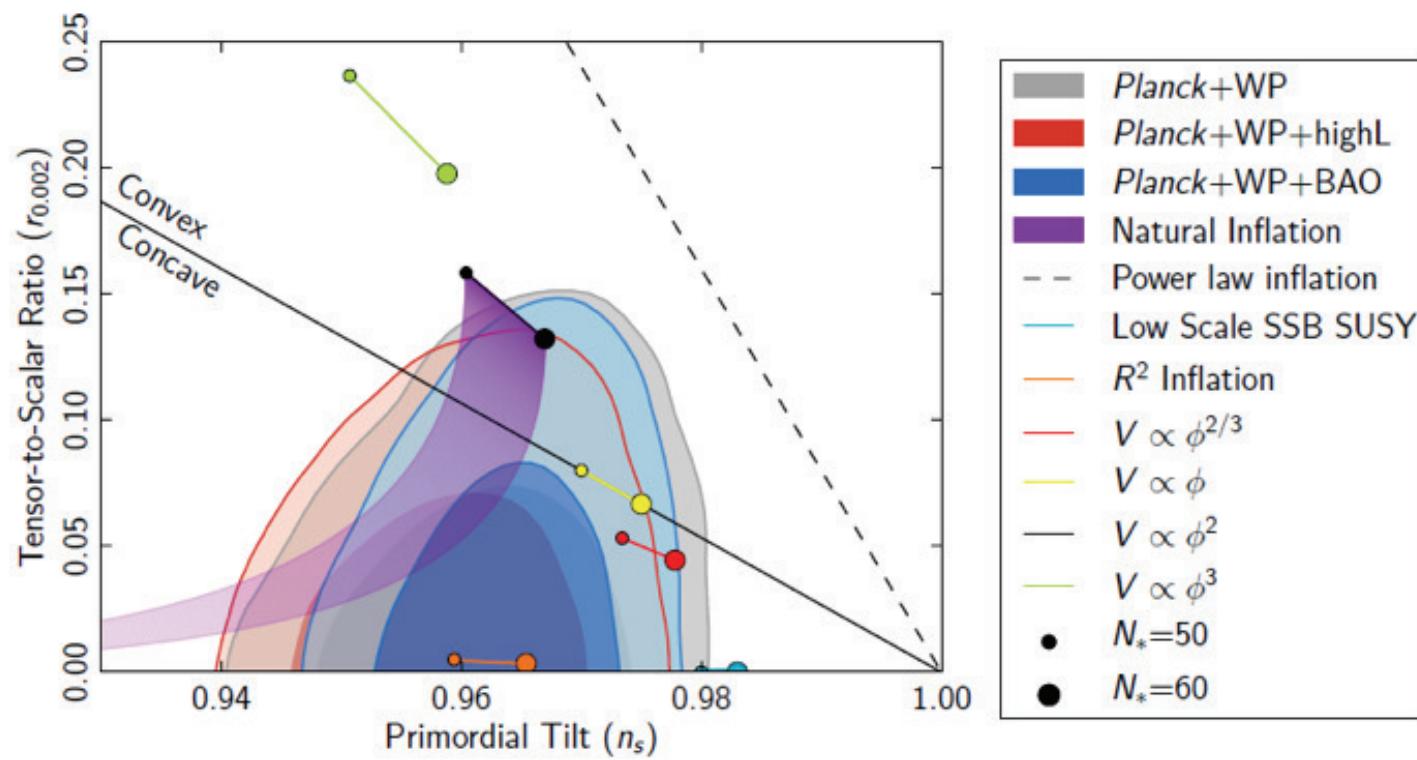
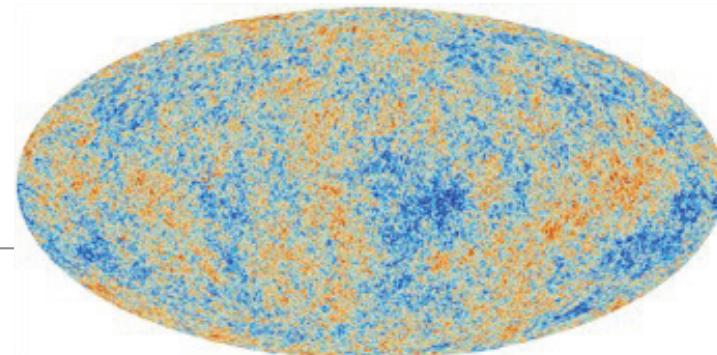
$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{1}{2}M^2 \left(1 + \xi \frac{\phi^2}{M^2}\right) R - \frac{1}{2}(\partial\phi)^2 - V(\phi)$$



V_{eff}



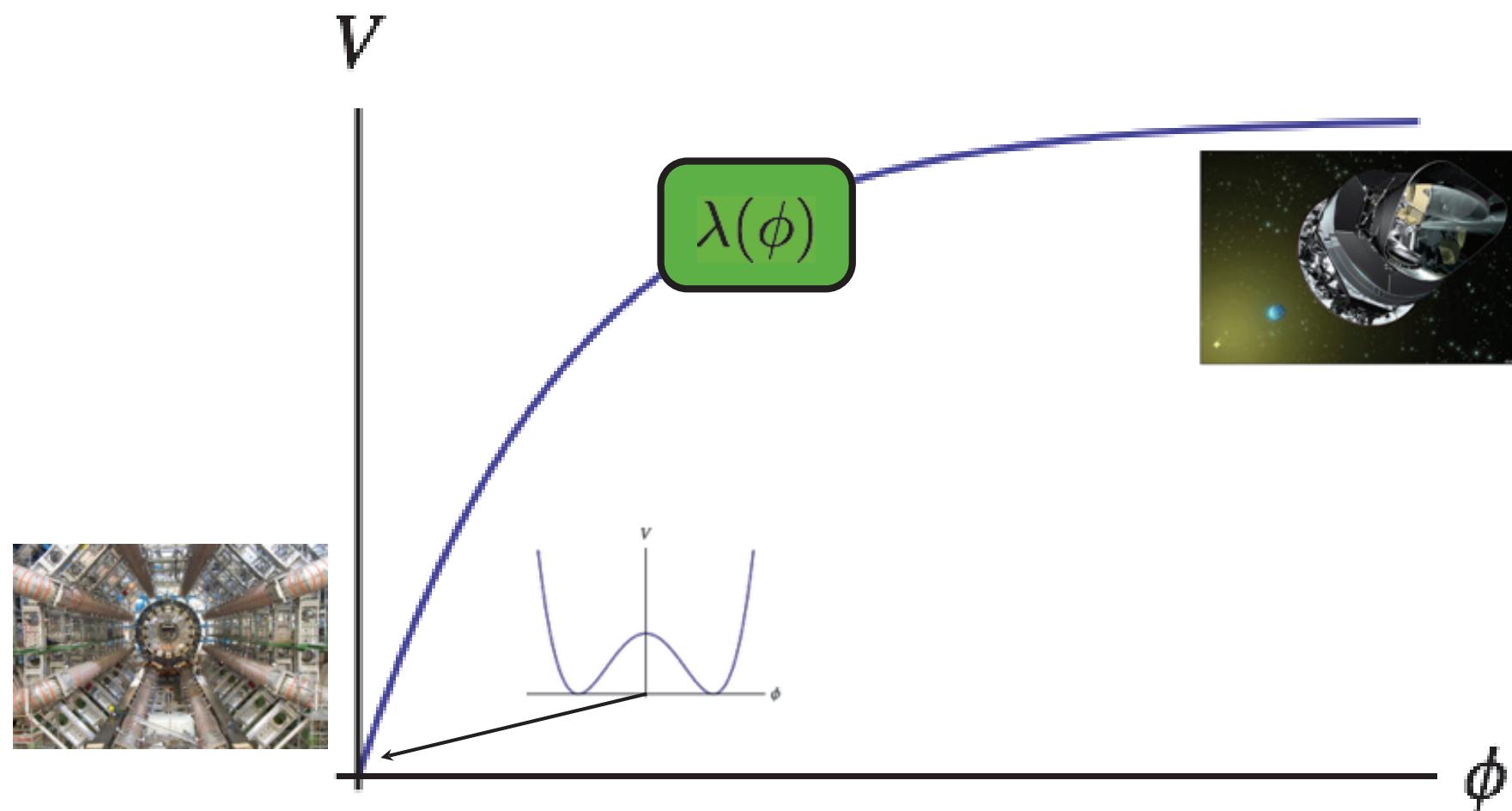
Planck results



Higgs inflation

Fakir '83, Salopek, Bond, Bardeen '89, Bezrukov & Shaposhnikov '08

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2} M^2 \left(1 + \xi \frac{\phi^2}{M^2} \right) R + |\partial\phi|^2 - V(\phi)$$



Higgs inflation

RGEs: $\mu \frac{\partial \lambda_i(\mu)}{\partial \mu} = \beta_i(\lambda)$

Bezrukov, Grubinov, Shaposhnikov
Barvinsky, Kamenshchik, Kiefer Starobinsky, Steinwachs
Simone, Hertzberg, Wilczek
etc.

Disagreement:

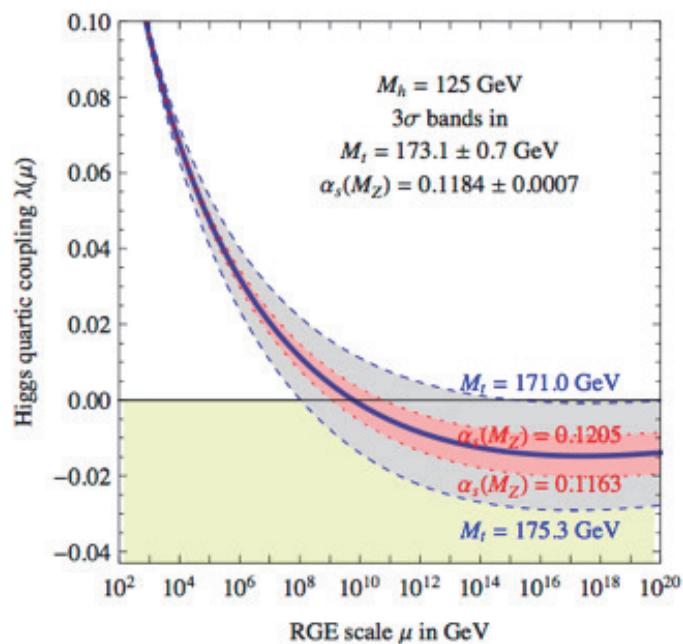
- goldstone bosons
- Einstein vs. Jordan frame



Potential problems with SM Higgs inflation

- stability bound

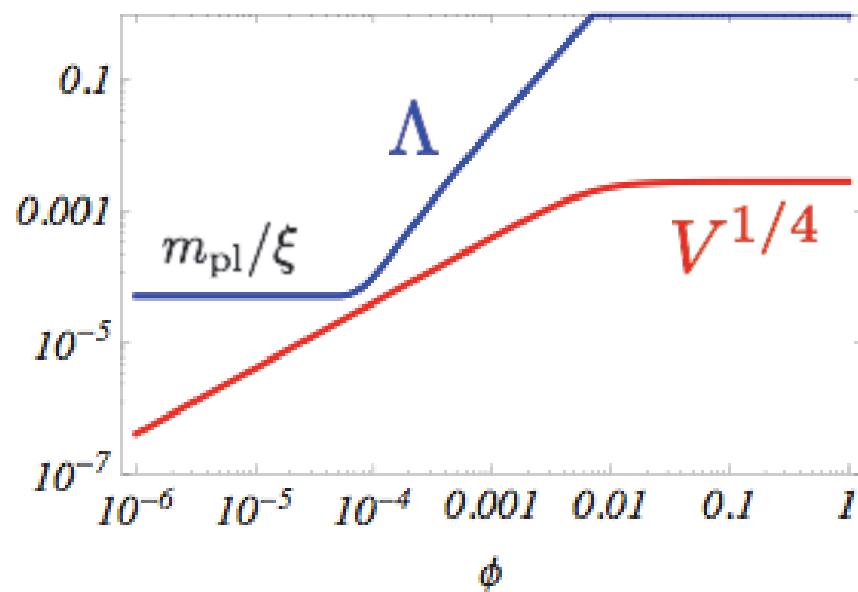
$$\lambda(\mu) < 0$$



Degrandi et al. '12, Bezrukov et al. '12

- unitarity bound

$$\mathcal{M}(\phi\phi \rightarrow \phi\phi) > 1$$



Ferrara et al. '11, Bezrukov et al. '11

Plan

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 - frame-independent approach

Conformal transformation

$$\frac{\mathcal{L}}{\sqrt{-g_J}} = -\frac{1}{2}M^2 \left(1 + 2\xi \frac{|\Phi|^2}{M^2} \right) R[g_J] - |\partial\Phi|^2 - V_J(|\Phi|)$$

Jordan frame $m_{\text{pl},J}^2 = M^2 \Omega^2(\Phi)$

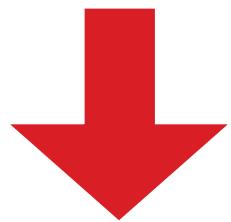
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$$\Omega^2$$

Jordan frame $m_{\text{pl},J}^2 = M^2 \Omega^2(\Phi)$

$$g_{\mu\nu}^E = \Omega^2 g_{\mu\nu}^J$$



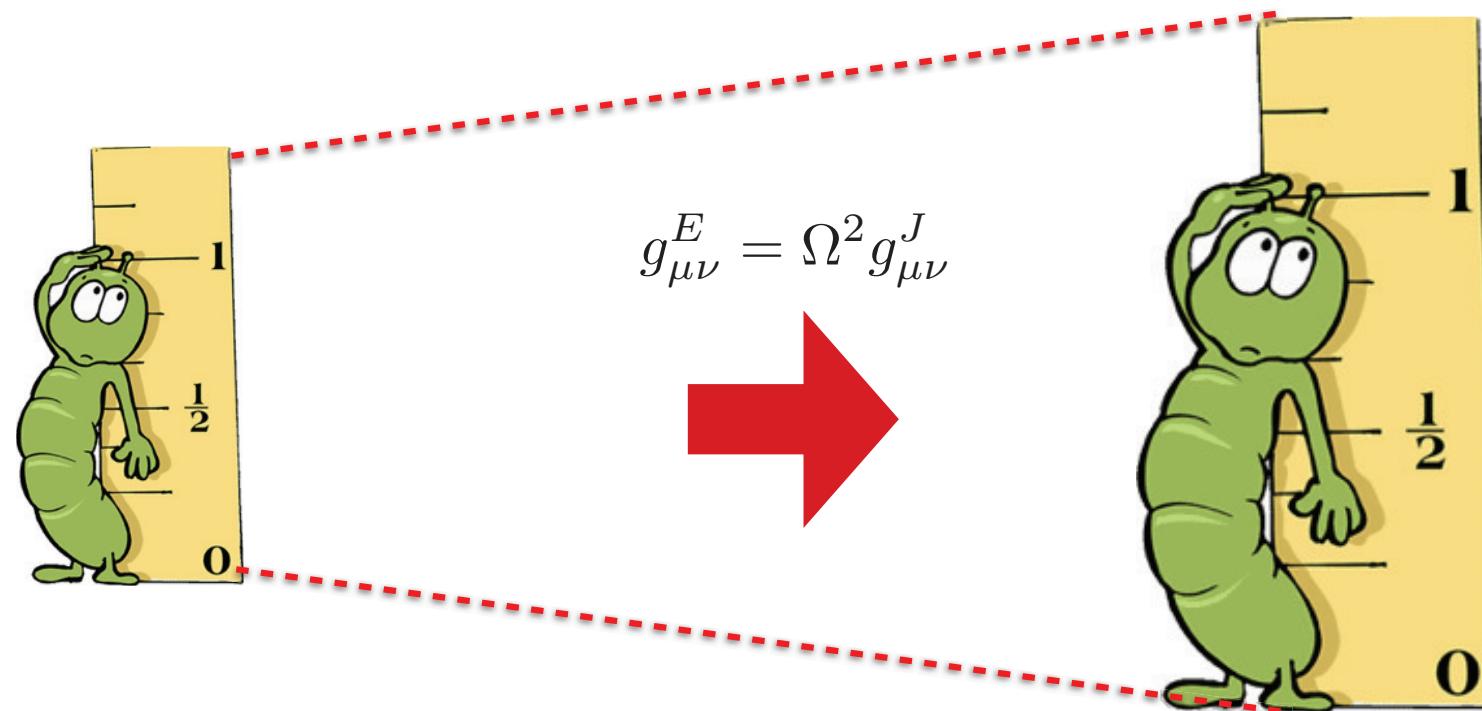
Einstein frame $m_{\text{pl},E}^2 = M^2$

$$\frac{\mathcal{L}}{\sqrt{-g_E}} = -\frac{1}{2}M^2 R[g_E] - \frac{1}{\Omega^2} |\partial\Phi|^2 - \frac{3\xi^2}{M^2 \Omega^4} |\partial|\Phi|^2|^2 - \frac{V_J(|\Phi|)}{\Omega^4}$$

Conformal transformation

R. Catena, M. Pietroni and L. Scarabello astro-ph/0604492;
E. Flanagan 2004

physics invariant: scales all mass/length scales



Conformal transformation

Wikipedia: “Despite the fact that these frames have been around for some time there is currently heated debate about whether either, both, or neither frame is a 'physical' frame which can be compared to observations and experiment.”

“the physical radius of the star, as measured in the physical Jordan frame”

“... embodies the two viable ranges $\xi < 0$ and $\xi < -1/6$ in the Einstein frame. In contrast, in the Jordan frame only the first of these is well-defined with a positive kinetic term..”

“This result leads us to conclude that the notion of adiabaticity is not invariant under conformal transformations.”

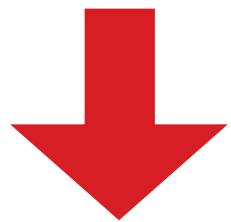
Conformal transformation

$$\frac{\mathcal{L}}{\sqrt{-g_J}} = -\frac{1}{2}M^2 \left(1 + 2\xi \frac{|\Phi|^2}{M^2} \right) R[g_J] - |\partial\Phi|^2 - V_J(|\Phi|)$$



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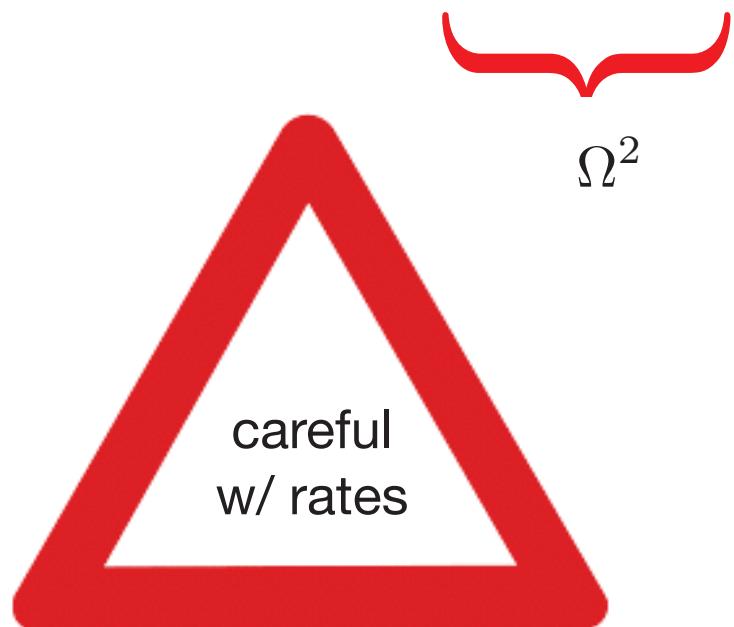
measurable, dimensionless quantities
invariant $m_{\text{pl},J}^2 ds_J^2 = m_{\text{pl},E}^2 ds_E^2$

Einstein frame $m_{\text{pl},E}^2 = M^2$

$$\frac{\mathcal{L}}{\sqrt{-g_E}} = -\frac{1}{2}M^2 R[g_E] - \frac{1}{\Omega^2} |\partial\Phi|^2 - \frac{3\xi^2}{M^2 \Omega^4} |\partial|\Phi|^2|^2 - \frac{V_J(|\Phi|)}{\Omega^4}$$

Conformal transformation

$$\frac{\mathcal{L}}{\sqrt{-g_J}} = -\frac{1}{2}M^2 \left(1 + 2\xi \frac{|\Phi|^2}{M^2} \right) R[g_J] - |\partial\Phi|^2 - V_J(|\Phi|)$$



Ω^2

Jordan frame $m_{\text{pl},J}^2 = M^2 \Omega^2(\Phi)$

measurable, dimensionless quantities
invariant

$$m_{\text{pl},J}^2 ds_J^2 = m_{\text{pl},E}^2 ds_E^2$$

Einstein frame $m_{\text{pl},E}^2 = M^2$

$$\frac{\mathcal{L}}{\sqrt{-g_E}} = -\frac{1}{2}M^2 R[g_E] - \frac{1}{\Omega^2} |\partial\Phi|^2 - \frac{3\xi^2}{M^2 \Omega^4} |\partial|\Phi|^2|^2 - \frac{V_J(|\Phi|)}{\Omega^4}$$

Plan

- Higgs inflation
- Frames
 - conformal transformation
 - quantization
 - running couplings in Higgs inflation
 - frame-independent approach

quantization

$$\frac{\mathcal{L}}{\sqrt{-g_J}} = -\frac{1}{2}M^2 \left(1 + \xi \frac{\phi^2}{M^2}\right) R[g_J] - \frac{1}{2}(\partial\phi)^2 - V + \dots \quad \text{Jordan frame}$$

$$\pi_i^J = \frac{\partial \mathcal{L}}{\partial \dot{\phi}_i^J} \quad [\phi_i^J(\vec{r}), \pi_j^J(\vec{r}')] = i\delta_{ij}\delta^{(3)}(\vec{r} - \vec{r}')$$

$$g_{\mu\nu}^E = \Omega^2 g_{\mu\nu}^J$$


$$\pi_i^E = \frac{\partial \mathcal{L}}{\partial \dot{\phi}_i^E} \quad [\phi_i^E(\vec{r}), \pi_j^E(\vec{r}')] = i\delta_{ij}\delta^{(3)}(\vec{r} - \vec{r}')$$

$$\frac{\mathcal{L}}{\sqrt{-g_E}} = -\frac{1}{2}M^2 R[g_E] - \frac{1}{2} \left(\frac{1}{\Omega^2} + \frac{6\xi^2\phi^2}{\Omega^4 M^2} \right) (\partial\phi)^2 - \frac{V}{\Omega^4} + \dots \quad \text{Einstein frame}$$

quantization

Non-minimal kinetic term

$$\mathcal{L} = \frac{1}{2} \gamma_{ij}(\phi^k) \partial_\mu \phi^i \partial^\mu \phi^j$$

- covariant approach

self-adjoint

$$[\phi^i(\vec{r}), \pi^j(\vec{r}')] = i\delta^{ij}\delta^{(3)}(\vec{r} - \vec{r}') \quad \longrightarrow \quad \pi^i = -i\nabla_{\phi^i}$$

Kamenshchik & Steinwachs '14

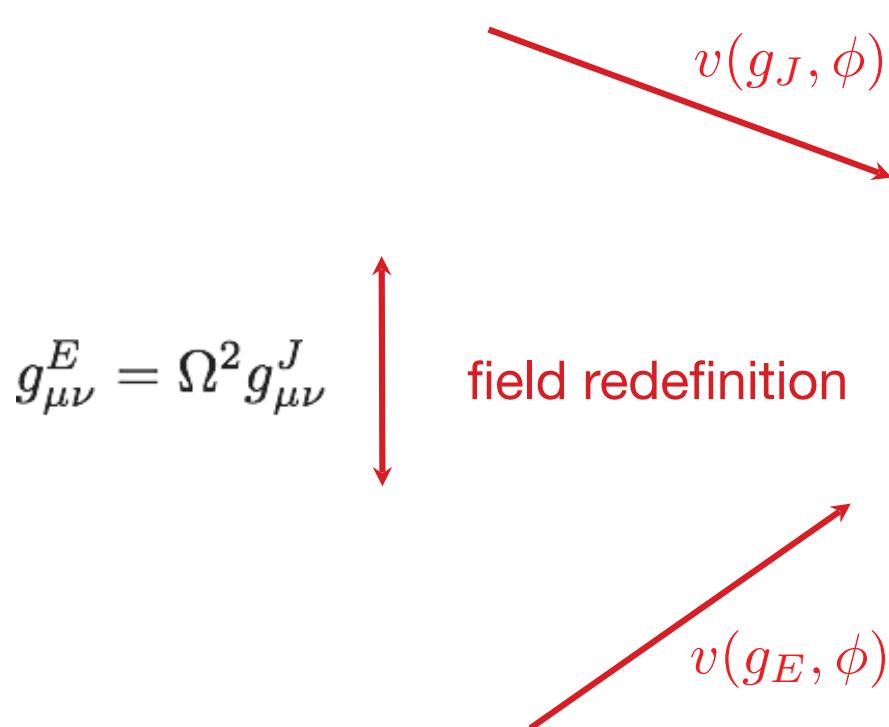
$$\phi^i(t, \vec{x}) = \phi_0(t) + \delta\phi(t, \vec{x})$$

not covariant

- diagonalize (quadratic) action

Conformal transformation

$$\frac{\mathcal{L}}{\sqrt{-g_J}} = -\frac{1}{2}M^2 \left(1 + \xi \frac{\phi^2}{M^2}\right) R[g_J] - \frac{1}{2}(\partial\phi)^2 - V + \dots \quad \text{Jordan frame}$$



$$\frac{\mathcal{L}}{\sqrt{-g_E}} = -\frac{1}{2}M^2 R[g_E] - \frac{1}{2} \left(\frac{1}{\Omega^2} + \frac{6\xi^2\phi^2}{\Omega^4 M^2} \right) (\partial\phi)^2 - \frac{V}{\Omega^4} + \dots \quad \text{Einstein frame}$$

what is done:

$$S = \int d^4x \left[\frac{1}{2}(\partial v)^2 - V \right]$$

$$[v(\vec{r}), \dot{v}(\vec{r}')] = i\delta^{(3)}(\vec{r} - \vec{r}')$$

unique quantization procedure

Kaiser '10, Prokopec & Weenink '12, '13,
White et al. '12, '13

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Error 1 – treat gravity as a classical background

$$\frac{\mathcal{L}}{\sqrt{-g_J}} = -\frac{1}{2}M^2 \left(1 + \xi \frac{\phi^2}{M^2}\right) \underline{R[g_J]} - \frac{1}{2}(\partial\phi)^2 - V + \dots \quad \text{Jordan frame}$$

$$\frac{\mathcal{L}}{\sqrt{-g_E}} = -\frac{1}{2}M^2 \underline{R[g_E]} - \frac{1}{2} \left(\frac{1}{\Omega^2} + \frac{6\xi^2\phi^2}{\Omega^4 M^2}\right) (\partial\phi)^2 - \frac{V}{\Omega^4} + \dots \quad \text{Einstein frame}$$

Error 1 – treat gravity as a classical background

$$\frac{\mathcal{L}}{\sqrt{-g_J}} = -\frac{1}{2}M^2 \left(1 + \xi \frac{\phi^2}{M^2}\right) \underline{R[g_J]} - \frac{1}{2}(\partial\phi)^2 - V + \dots \quad \text{Jordan frame}$$

error large:

- Higgs/GB not suppressed
- wrong β_ξ



error sub-leading

$$m_v^2 = -2H^2(2 - 3\eta - \epsilon + 6\epsilon)$$



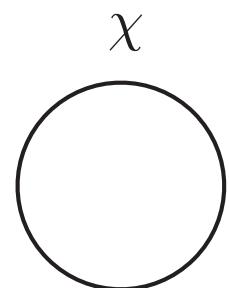
$$\frac{\mathcal{L}}{\sqrt{-g_E}} = -\frac{1}{2}M^2 \underline{R[g_E]} - \frac{1}{2} \left(\frac{1}{\Omega^2} + \frac{6\xi^2\phi^2}{\Omega^4 M^2} \right) (\partial\phi)^2 - \frac{V}{\Omega^4} + \dots \quad \text{Einstein frame}$$

Error 2 – field dependent cutoff

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\chi)^2 - \frac{y}{2}\phi^2\chi^2 - \frac{\lambda}{4}\phi^4$$

Coleman-Weinberg 1-loop correction:

$$V_{\text{eff}} = \frac{\lambda}{4}\phi_0^4 - \frac{1}{(8\pi)^2}m_\chi(\phi_0)^4 \ln\left(\frac{\Lambda^2}{m_\chi^2}\right)$$



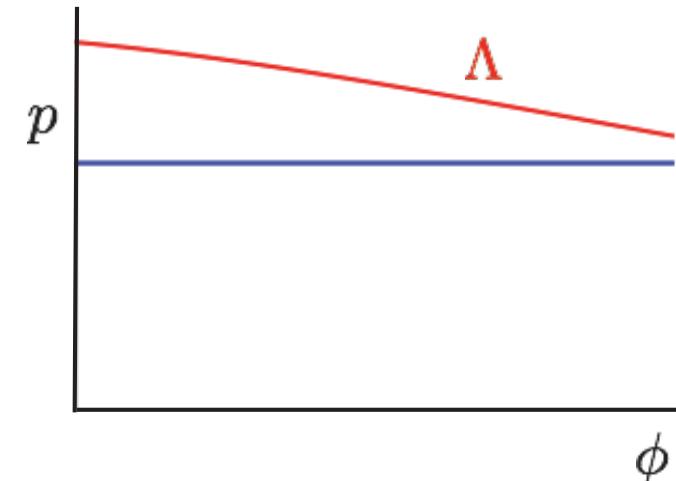
Error 2 – field dependent cutoff

UV dependence?

$$\Lambda_E = \frac{\Lambda_J}{\Omega}$$



Jordan frame

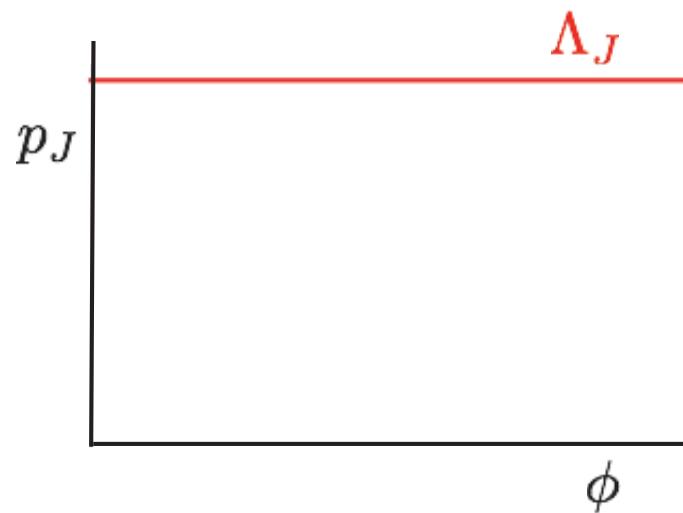


Einstein frame

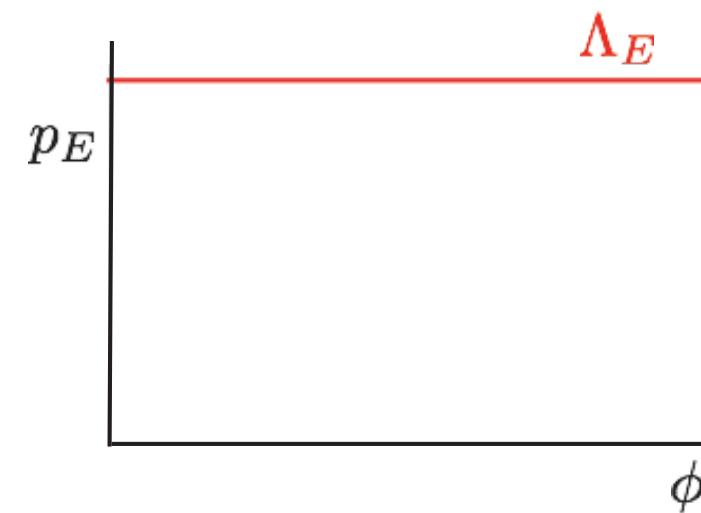
Error 2 – field dependent cutoff

resolution: mass ratios invariant

$$\delta V_J \propto m_J^4 \ln(\Lambda_J^2/m_J^2) \quad \Rightarrow \quad \delta V_E \propto m_E^4 \ln(\Lambda_E^2/m_E^2)$$



Jordan frame



Einstein frame

Error 2 – field dependent cutoff

$$\beta_\lambda = \partial\lambda/\partial\ln\mu$$

$$\tau = \chi/\Omega$$

$$\begin{aligned}\frac{\mathcal{L}}{\sqrt{-g_J}} = & \frac{1}{2}(1 + \xi\phi^2)R + \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\chi)^2 \\ & - \frac{y^2}{2}\phi^2\chi^2 - \frac{\lambda}{4}\phi^4\end{aligned}$$

$$\begin{aligned}\frac{\mathcal{L}}{\sqrt{-g_E}} = & \frac{1}{2}R + \frac{1}{2}\gamma(\partial\phi)^2 + \frac{1}{2}(\partial\tau)^2 \\ & - \frac{y^2}{2\Omega^2}\phi^2\tau^2 - \frac{\lambda}{4\Omega^4}\phi^4\end{aligned}$$

$$V_J = \frac{\lambda}{4}\phi^4 + \frac{\delta\lambda}{4}\phi^4 - cm_\chi^4 \ln\left(\frac{\Lambda_J^2}{m_\chi^2}\right)$$

1-to-1

$$V_E = \frac{\lambda}{4\Omega^4}\phi^4 + \frac{\delta\lambda}{4\Omega^4}\phi^4 - cm_\tau^4 \ln\left(\frac{\Lambda_E^2}{m_\tau^2}\right)$$

$$\lambda(\mu_J) = \frac{4}{\phi^4} V_J \Big|_{m_\chi^2 = \mu_J^2}$$



$$\lambda(\mu_E) = \frac{4\Omega^4}{\phi^4} V_E \Big|_{m_\tau^2 = \mu_E^2}$$

$$V_J = \frac{\phi^4}{4} \left[\lambda - 4cy^4 \ln\left(\frac{\mu_J^2}{m_\chi^2}\right) \right]$$

$$g_{\mu\nu}^E = \Omega^2 g_{\mu\nu}^J$$

$$V_E = \frac{\phi^4}{4\Omega^4} \left[\lambda - 4cy^4 \ln\left(\frac{\mu_E^2}{m_\tau^2}\right) \right]$$

$$\beta_\lambda = 8cy^4, \quad \lambda(m_{EW}^J) = \lambda_0, \quad m_{EW}^J < \mu_J < m_\chi$$

$$\beta_\lambda = 8cy^4, \quad \lambda(m_{EW}^E) = \lambda_0, \quad m_{EW}^E < \mu_E < m_\tau$$

$$V_J^{\text{impr}}(m_\chi) = \frac{\lambda(m_\chi)}{4}\phi^4$$

$$V_E^{\text{impr}}(m_\tau) = \frac{\lambda(m_\tau)}{4\Omega^4}\phi^4$$

Jordan frame

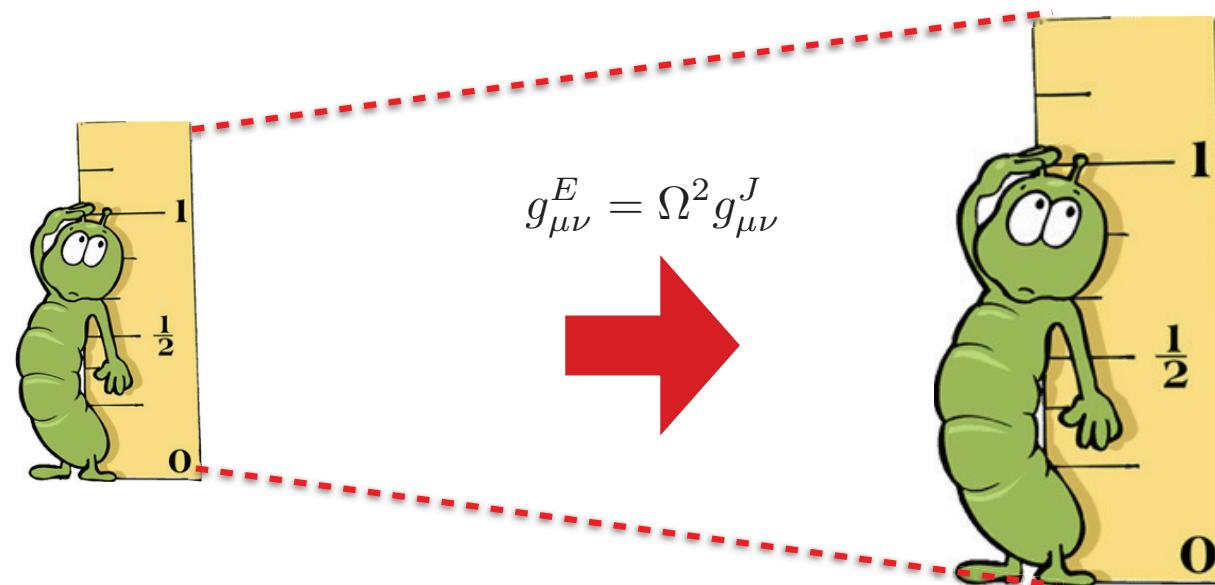
Einstein frame

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Frame-independent action

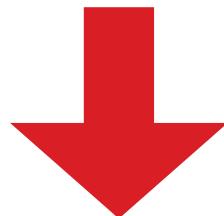
dimensionless quantities invariant: **write action in planck units**



Frame-independent action

$$S = -\frac{1}{2} \int d^4x \sqrt{-g_J} [M^2 \Omega^2 R[g_J] + (\partial\phi)^2 + 2V_J]$$

Jordan frame $m_J^2 = M^2 \Omega^2$



$$g_{\mu\nu}^E = \Omega^2 g_{\mu\nu}^J$$

$$m_J^2 ds_J^2 = m_J^2 [-N_J^2 dt^2 + a_J^2 dx^2] = m_E^2 [-N_E^2 dt^2 + a_E^2 dx^2] = m_E^2 ds_E^2$$

Einstein frame $m_E^2 = M^2$

$$S = -\frac{1}{2} \int d^4x \sqrt{-g_E} \left[M^2 R[g_E] + \gamma_{\phi\phi} (\partial\phi)^2 + 2 \frac{V_J}{\Omega^4} \right]$$

Frame-independent action

$$m_J^2 ds_J^2 = m_J^2 [-N_J^2 dt^2 + a_J^2 dx^2] = m_E^2 [-N_E^2 dt^2 + a_E^2 dx^2] = m_E^2 ds_E^2$$

dimensionless quantities:

$$\bar{N} = m_i N_i, \quad \bar{a} = m_i a_i, \quad i = J, E$$

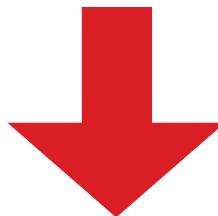
$$\sqrt{-\bar{g}} = \sqrt{-g_i} m_i^4$$

$$\bar{H} = \frac{\bar{a}'}{\bar{a}} = \frac{1}{\bar{a}} \frac{\partial_t \bar{a}}{\bar{N}} = \frac{1}{a_i m_i} \frac{1}{m_i N_i} \partial_t (a_i m_i)$$

$$\bar{\phi} = \phi_i / m_i$$

Frame-independent action

$$S = -\frac{1}{2} \int d^4x \sqrt{-g_E} M^4 \frac{[M^2 R[g_E] + (\partial\phi)^2 + 2\frac{V_J}{\Omega^4}]}{M^4}$$



rewrite in dim. quantities

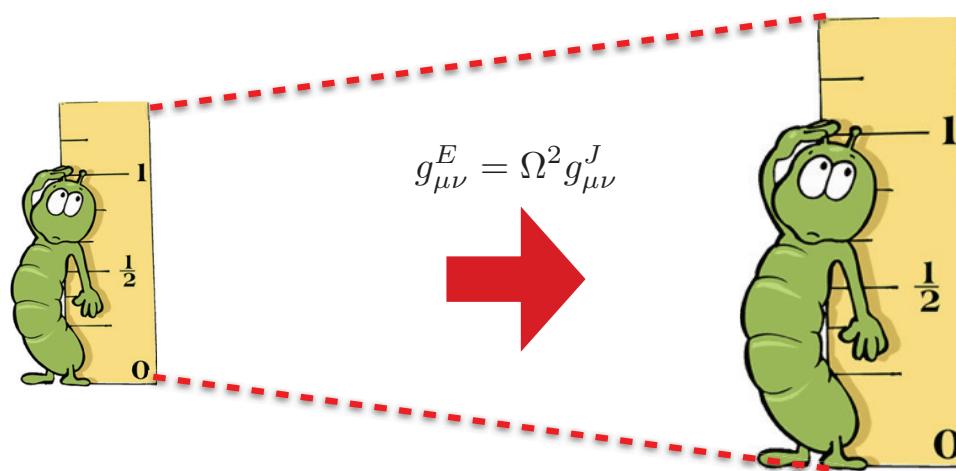
$$S = \int d^4x \sqrt{-\bar{g}} \left(\frac{1}{2} \bar{R} - \frac{1}{2} \bar{S}_{ab} \bar{g}_{\mu\nu} \nabla^\mu \bar{\phi}^a \nabla^\nu \bar{\phi}^b - \bar{V} \right)$$

invariant under conformal transformation

$$\bar{S}_{ab} = \delta_{ab} + \bar{\phi}_a \bar{\phi}_b \left(\frac{\xi(8 - \xi \bar{\phi}^2)}{(1 - \xi \bar{\phi}^2)^2} \right)$$

Conclusions

- Higgs inflation disagreement on quantum corrections
- Frames physics frame independent



Higgs inflation

RGEs: $\mu \frac{\partial \lambda_i(\mu)}{\partial \mu} = \beta_i(\lambda)$

Bezrukov, Grubinov, Shaposhnikov
Barvinsky, Kamenshchik, Kiefer Starobinsky, Steinwachs
Simone, Hertzberg, Wilczek
etc.

Disagreement:

- goldstone bosons
- Einstein vs. Jordan frame

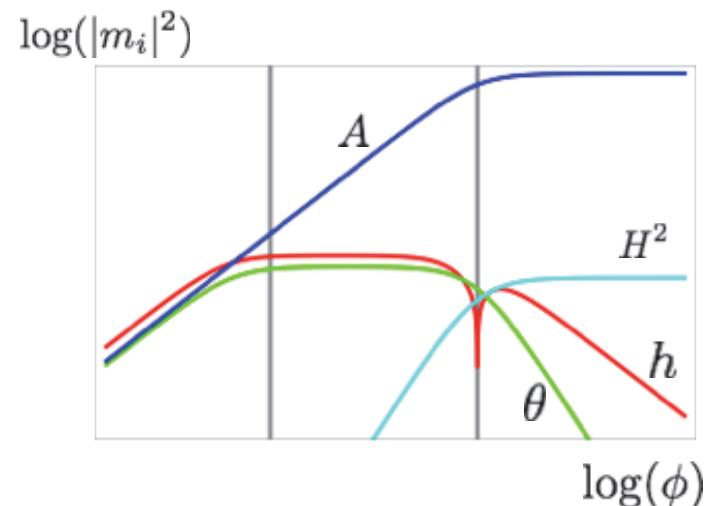


Goldstone bosons

GB decouples

$$\delta V = \frac{1}{(8\pi)^2} \sum_i (-1)^{F_i} m_i^4 \ln \left(\frac{m_i^2}{\mu^2} \right)$$

Coleman & Weinberg '73



GB does not decouple

$$\Sigma_\psi = \text{Diagram with two arrows and a dashed arc} + \text{Diagram with a wavy line and a fermion loop} + \dots$$

$\mathcal{L} \supset \bar{\psi}_L i\gamma \cdot D\psi_L + \bar{\psi}_R i\gamma \cdot D\psi_R$

Lagrangian: covariant formulation

$$\delta\phi^a = Q^a - \frac{1}{2!}\Gamma_{bc}^a Q^b Q^c + \frac{1}{3!} (\Gamma_{be}^a \Gamma_{cd}^e - \Gamma_{bc,d}^a) Q^b Q^c Q^d + \dots$$

covariant!

expand

$$\phi^a = (\phi_0(t) + \delta\phi(x, t), \delta\theta(x, t))$$

not covariant!

