Nuclear Dark Matter

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Large Composite Dark Matter States

Standard model: conserved baryon number, attractive interactions lead to a multitude of large, stable bound states (nuclei!).

What if a similar thing happens for dark matter?

Not so implausible: strong coupling is a very reasonable way for the DM mass scale to be generated

Can states of large dark nucleon number be built up in the early universe?

Possible Consequences

- Very heavy DM $\gtrsim 10^3\,\mathrm{TeV}$ (usually disfavoured for freeze-out, and in asymmetric DM models)
- Different sized states important at different stages in cosmological history- changing 'properties'
- Coherent enhancement of interactions, e.g. direct detection vs. collider limits
- Structure at parametrically smaller energy scales than constituents (overall size, maybe shell structures) leads to form factors in direct detection.
- Inelastic modes: 'high energy' fissions/ fusions at typical binding energy scale, collective excitations at parametrically lower energies $\sim \frac{\epsilon}{R}$

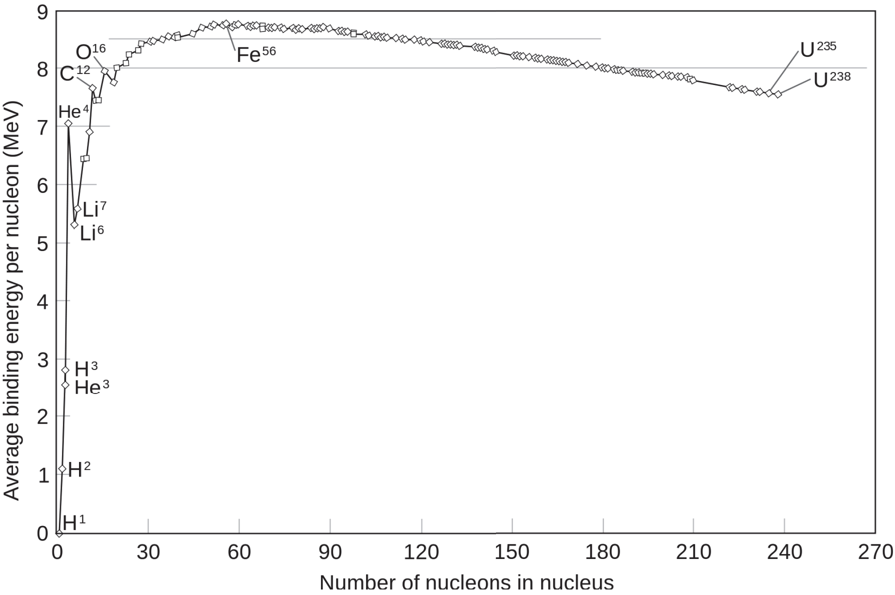
All of these depend on the spectrum of dark matter states

This talk

1. Dark nucleosynthesis
2. Solutions to the aggregation equations
3. Effect on direct detection
4. Astrophysics & Phenomenology

Try to be as model independent as possible

Standard Model



Dark Sector

Assume:

- Approximately conserved nucleon number (A), asymmetric dark matter simplest (symmetric also interesting)
- Relatively short-range strong 'nuclear' binding force
- Hard core repulsion
- No dark analogy of the photon

Then binding energy per dark nucleon saturates at large A:

$$\frac{B_A}{A} = \alpha - \frac{\beta}{A^{1/3}},$$

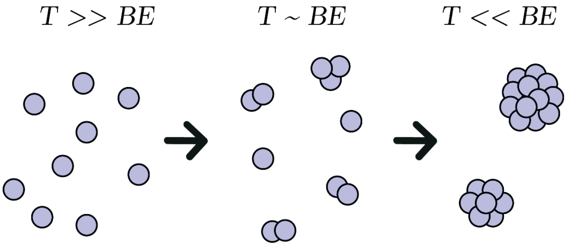
Small A: states/ interactions have complicated properties
Large A: cross sections scale geometrically, constant density

Dark Nucleosynthesis

Free energy $F = E - TS$

large T $\implies$ everything dissociated

small T $\implies$ large states favoured



Freeze out of Dissociations

Equilibirum number densities

$$\frac{\tilde{n}_A}{\tilde{n}_{A-k}\tilde{n}_k} = \frac{g_A}{g_k g_{A-k}} \left(\frac{2\pi}{m_A T} \frac{m_A^2}{m_k m_{A-k}} \right)^{3/2} e^{(B_A - B_k - B_{A-k})/T} ,$$

Fusion dominates once T drops enough that

$$\frac{n_k n_{A-k}}{n_A} \gg \frac{\tilde{n}_k \tilde{n}_{A-k}}{\tilde{n}_A}$$

$$\Leftrightarrow \quad n_0 \left(2\pi/(m_1 T)\right)^{3/2} e^{\Delta B/T} \gg 1$$

- DM is fairly dilute so $n_0 \left(2\pi/(m_1 T)\right)^{3/2} \ll 1$
- Transition at $T \ll \Delta B$ (around $1\,\mathrm{MeV}$ for SM)
- Exponential dependence, transitions in a small fraction of a Hubble time- much smaller than time fusions occur for.

Freeze-out of fusions

Peaked mass distribution: fusions always of form $A + A \to 2A$

$$\frac{\Gamma}{H} \sim \frac{\sigma_1 v_1 n_0}{H} A^{2/3} A^{-1/2} A^{-1} = \frac{\sigma_1 v_1 n_0}{H} A^{-5/6} ,$$

$$\frac{\sigma_1 v_1 n_0}{H} \sim 2{\times}10^7 \left(\frac{1\,\mathrm{GeV\,fm}^{-3}}{\rho_b} \right)^{2/3} \left(\frac{g_\star(T)}{10.75} \right)^{1/2} \left(\frac{m_1}{1\,\mathrm{GeV}} \right)^{-5/6} \left(\frac{T}{1\,\mathrm{MeV}} \right)^{3/2} ,$$

If dissociation stops being important around $T = 1\,\mathrm{MeV}$ maximum mass:

$$(2 \times 10^7)^{6/5} m_1 \sim 5 \times 10^8\,\mathrm{GeV}$$

$$\mathrm{radius} \sim 480\,\mathrm{fm}$$

Increasing mass scales leads to smaller DM masses, $T \mapsto \lambda T$, $\rho_b \mapsto \lambda^4 \rho_b$, etc. :

$$m_{\mathrm{fo}} \mapsto \lambda^{-7/5} m_{\mathrm{fo}}$$

Significant proportion of mass in small states: last to freeze out are $A + 1 \to (A + 1)$
Higher number density of small A for given mass fraction, need more fusions for given increase in A, but small A have greater velocities: *enhancement*

$$\langle \sigma v \rangle_{A+1} = \delta \sigma_1 A^{2/3} v_1,$$

$$\Gamma \sim \langle \sigma v \rangle n_1 \frac{1}{A} \sim (1 - \alpha) \delta \sigma_1 v_1 n_0 A^{2/3} A^{-1} \ ,$$

$1 - \alpha$ is the mass fraction in small A

$$A_{\mathrm{fo}} \sim 7 \times 10^{19} \left((1 - \alpha) \delta \frac{\sigma_1 v_1 n_0}{H} \Big/ 2 \times 10^7 \right)^3,$$

Aggregation equations

Boltzmann equation for k-DN number density, $n_k(t)$, is

$$\frac{dn_k(t)}{dt}+3H(t)n_k(t) = -\sum_{j=1}^{\infty}\langle\sigma v\rangle_{j,k}n_j(t)n_k(t)+\frac{1}{2}\sum_{i+j=k}\langle\sigma v\rangle_{i,j}n_i(t)n_j(t)\,,$$

Switch to yields $Y_k \equiv n_k/s$, and define a new 'time' variable

$$\frac{dw}{dt} = Y_0\sigma_1 v_1(t)s(t) = n_0(t)\sigma_1 v_1(t)\,,$$

Gives

$$\frac{dy_k}{dw} = -y_k\sum_j K_{j,k}y_j + \frac{1}{2}\sum_{i+j=k} K_{i,j}y_iy_j\,,$$

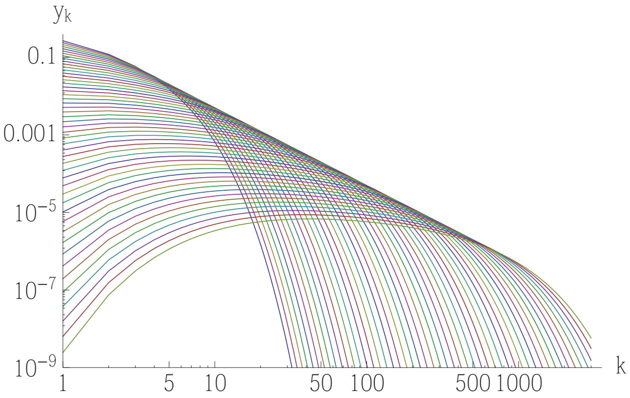
where $K_{i,j}$ encode relative rates of different fusion processes, $y_k = \frac{Y_k}{Y_0}$

Scaling Solution

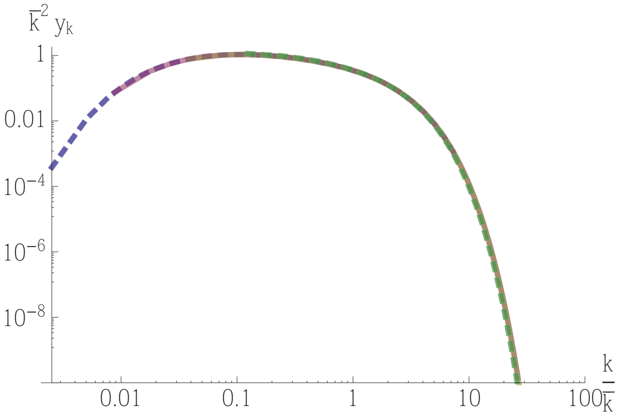
Cross-section $\sim$ $(area + area)\, v_{rel}$ means

$$K_{i,j} = (i^{2/3} + j^{2/3}) \left(\frac{1}{i^{1/2}} + \frac{1}{j^{1/2}} \right) ,$$

Number distribution at equally spaced values in $\log w$:

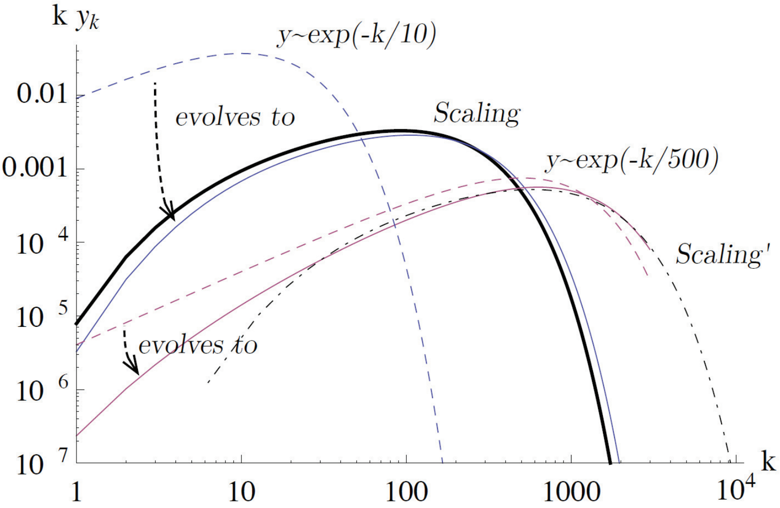


Shape stays the same, average size increases, $s(w) \sim w^{6/5}$



Attractor solution, depending only on large-k behaviour of kernel- reach this form (eventually) independent of initial conditions, small-k kernel.

Varying Initial Conditions



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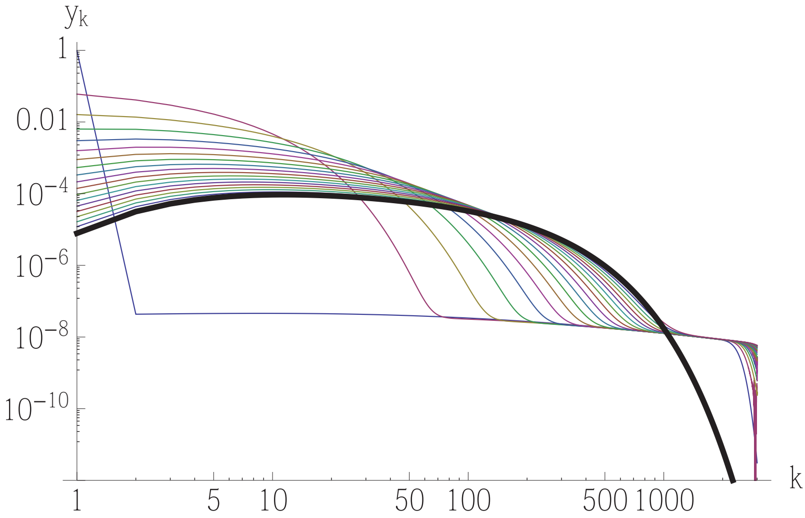
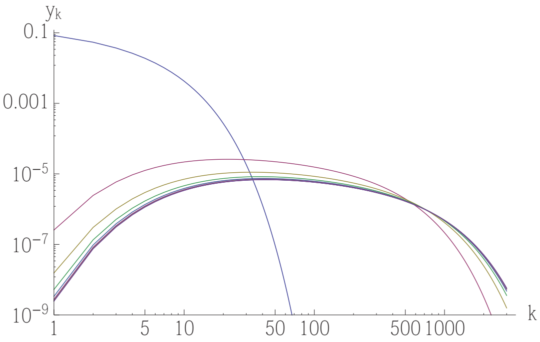


Real Time Behaviour

$$\frac{dw}{dT} \simeq -\left.\frac{n_0 \sigma_1 v_1}{H}\right|_{T_0} \left(\frac{T}{T_0}\right)^{1/2} \frac{1}{T_0},$$

$$w(T) \simeq \frac{2}{3} \left.\frac{n_0 \sigma_1 v_1}{H}\right|_{T_0} \left(1 - \left(\frac{T}{T_0}\right)^{3/2}\right)$$

Most of build-up within one Hubble time. Distribution at half e-folding real times:

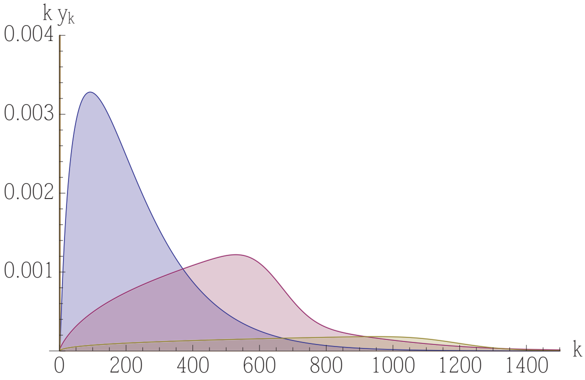


What if there's a bottle-neck at small numbers?

If $K_{i,j}$ for small i, j is low enough, and w_{max} is small enough, never reach scaling regime

$$\frac{dk}{dw} \sim K_{1,k} y_1 \sim k^{2/3}$$

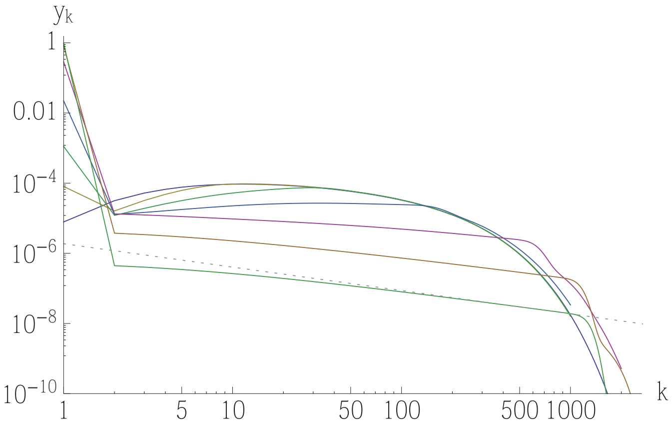
$$k_{max} \sim \left(\int dw \, y_1 \right)^3$$



Mass distribution at $w = 25$ for $K_{1,1} = 1, 10^{-4}, 10^{-5}$

Power-law distribution

Number distribution: $w = 25$ for $K_{1,1} = 1, 10^{-1}, ..., 10^{-6}$



If we have a bath of small-number states throughout:

$$k \sim (w_{max} - w_{inj})^3 \Rightarrow -\frac{dw_{inj}}{dk} \sim k^{-2/3} \Rightarrow y_k \sim k^{-2/3}$$

Direct detection

- Interactions with SM coherently enhanced by $A^2 \times$form factor
Number density $\sim \frac{1}{A}$, so total direct detection rate enhanced by A

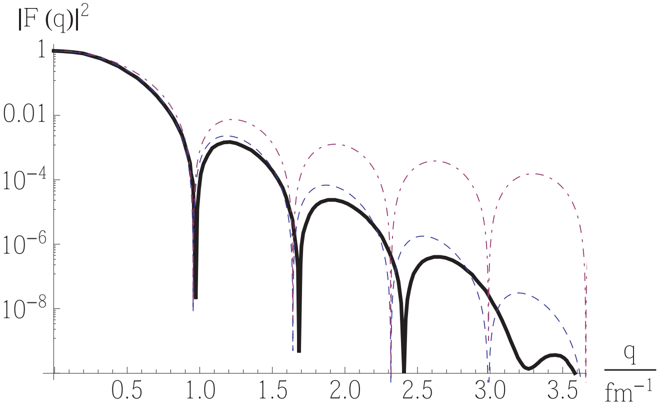
- For given direct detection rate, production at colliders etc. suppressed.

- Constraints on early-universe annihilations to SM stronger than usual

Form factors

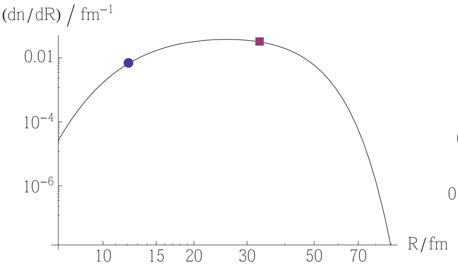
If $R_{DM} > (\Delta p)^{-1}$ probe dark matter form factor, e.g. spherical Bessel

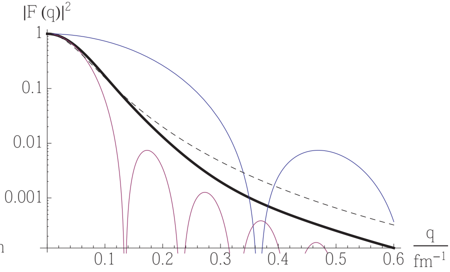
$$F(q) = \frac{3 j_1(qR)}{qR} e^{-q^2 s^2/2},$$



Scattering cross section

$$\frac{dR}{dE_R} = F_N(q)^2 \sum_i g_i(v_{\min}(\mu_{i,N})) \frac{n_i}{2\mu_{i,n}^2} \frac{|C_i|^2}{|C_n|^2} \sigma_{i,n} F_{X,i}(q)^2,$$





Recoil Spectra For Size Distribution

$g(v_{\text{min}}) = \int_{v > v_{\text{min}}} d^3\mathbf{v} \frac{f(\mathbf{v})}{v}$: by choosing $f(\mathbf{v})$ can make g any non-increasing function of v_{min}

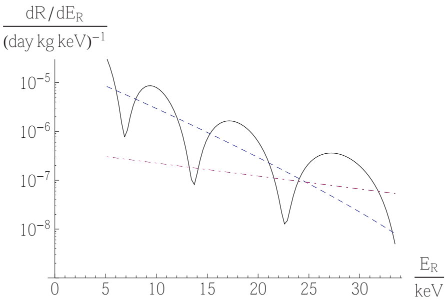
Data from multiple detectors lifts the degeneracy, since $q(v_{\text{min}}) = 2\mu_{XN} v_{\text{min}}$, changing μ_{XN} by changing m_N will change the range of $F_X(q)$

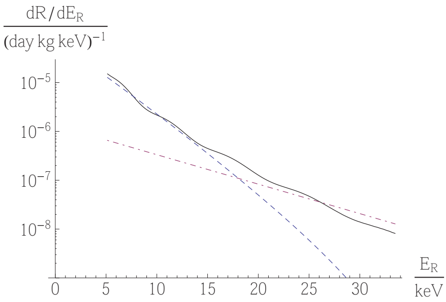
Effective exposures 1.0 ton years in multiple detectors could allow determination of the average DM size to around 25% accuracy. (related work: Cherry, Frandsen, and Shoemaker)

Detectability of Rising Recoil Spectrum

E.g. if sharply peaked number distribution due to binding energy curve turning over

Germanium vs. Xenon





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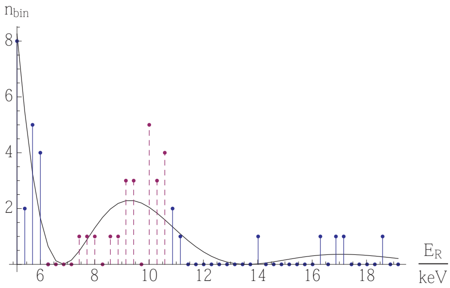
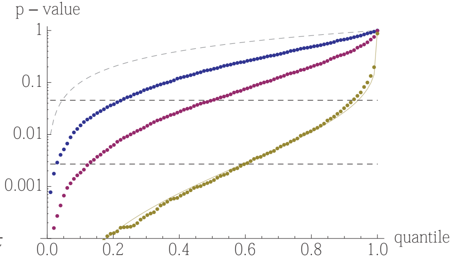




Identify the interval worst fit by a non rising distribution.

Inelastic Modes

Compressional: speed of sound $c_c \sim \sqrt{\frac{K}{\rho}}$, where K parameterises compressibility

$$\delta E_c \sim \sqrt{\frac{K}{\rho}}\frac{1}{R} \ . \tag{1}$$

Surface: $c_s \sim \sqrt{\frac{\sigma k}{\rho}}$, where σ is the surface tension

$$\delta E_s \sim \sqrt{\frac{\sigma}{\rho}}\frac{1}{R^{3/2}} \ . \tag{2}$$

same microscopic force so $\sigma \sim R_1 K$, and

$$\frac{\delta E_s}{\delta E_c} \sim A^{-1/6} \ . \tag{3}$$

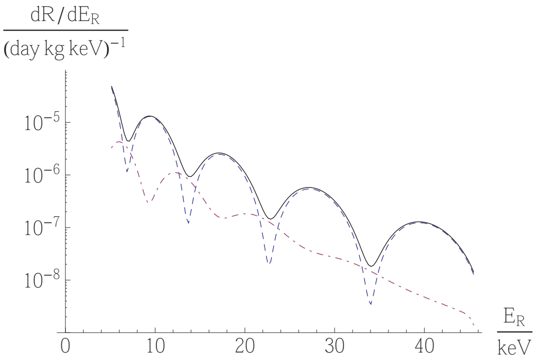
(Single constituent excitations $\omega \sim A^{-1/3}/R$ but cross section not enhanced)

Inelastic Modes

Form factors can be computed, e.g. one phonon surface mode

$$F(q) = \frac{3A}{4\pi} i^l (2l+1)^{1/2} \epsilon_l j_l(qR) \tag{4}$$

$$v_{min} = \frac{1}{\sqrt{2m_N E_R}} \left(\frac{m_N E_R}{\mu_{XN}} + \delta \right) \tag{5}$$



Phenomenology

- Usual constraints of getting rid of abundances of other hidden sector states and symmetric component (large energy injections to SM safest before BBN)

- New feature release of binding energy from fusions.

- Most binding energy released during dark nucleosynthesis: if inject significant energy to SM, safest before BBN

Self interaction

Observational limit

$$\sigma_{XX}/m_X \lesssim 1\,\mathrm{barn}/\,\mathrm{GeV} \tag{6}$$

NDM:

$$\frac{\sigma_{AA}}{m_A} \simeq \frac{0.05\,\mathrm{barn}}{\mathrm{GeV}} A^{-1/3} \left(\frac{1\,\mathrm{GeV}}{m_1}\right)^{1/3} \left(\frac{1\,\mathrm{GeV\,fm}^{-3}}{\rho_b}\right)^{2/3}, \tag{7}$$

Self interaction cross sections constrain dark nucleons $\gtrsim 10\,\mathrm{MeV}$:

Post-nucleosynthesis interactions

More exotic possibilities:

- Endothermic, e.g. scatter to excited states.
  At $\rho_{DM} \sim 10\, A^{1/3}\,\mathrm{GeV cm}^{-3}$ most DN interact (occurs at $r \sim 0.5 A^{-1/3}\,\mathrm{kpc}$ in NFW), run away process?

- Exothermic- get a velocity kick. Possibly clear out high density regions, modify structure on small scales.

Post-nucleosynthesis interactions

Proportion of mass density injected from fusions

$$\langle \sigma v \rangle n_A t_{\rm sys} \frac{\Delta BE}{M_A} \sim 10^{-3} A^{-2/3} \frac{\Delta BE}{A^{2/3} 0.01 m_1} \left\{ \frac{\rho_{\rm DM}}{0.3\,{\rm GeV\,cm^{-3}}} \right.$$

$$\left. \left(\frac{1\,{\rm GeV\,fm^{-3}}}{\rho_b} \right)^{2/3} \left(\frac{1\,{\rm GeV}}{m_1} \right)^{1/3} \left(\frac{v}{10^{-3}} \right) \left(\frac{t_{\rm sys}}{10\,{\rm Gyr}} \right) \right\}$$

(c.f. usual DM, s-wave annihilations s-wave annihilations is

$$\langle \sigma v \rangle n_X t_{\rm gal} \sim 3 \times 10^{-8} \left(\frac{100\,{\rm MeV}}{m_X} \right) \left(\frac{\langle \sigma v \rangle_X}{\rm pb} \right) \left(\frac{\rho_{\rm DM}}{0.3\,{\rm GeV\,cm^{-3}}} \right) ,$$

)

Capture in Astrophysical Objects

Solar capture rate

$$C_{\odot} \simeq 3 \times 10^{19}\,\mathrm{sec}^{-1} \left(\frac{\sigma_{Xn}}{10^{-8}\,\mathrm{pb}}\right) \left(\frac{1\,\mathrm{TeV}}{m_X}\right)^2 ,$$

(fraction of incident DM captured $\sim 3 \times 10^{-7} \left(\frac{\sigma_{Xn}/m_X}{10^{-8}\,\mathrm{pb}/\,\mathrm{TeV}}\right)$.)

May settle into an isothermal distribution: $\rho_X \sim e^{-r/(2r_\star^2)}$, with

$$r_\star = \left(\frac{3\,T_\odot}{4\pi G m_X \rho_\odot}\right)^{1/2} \simeq 2\times10^{-3} R_\odot \left(\frac{\mathrm{TeV}}{m_X}\right)^{1/2} \left(\frac{T_\odot}{10^7\,\mathrm{K}}\right)^{1/2} \left(\frac{150\,\mathrm{g\,cm}^{-3}}{\rho_\odot}\right)^{1/2}$$

Most extreme possibility is a run-away process ending with almost all the DN in a large central state.

But this has constant density, and doesn't annihilate (unless you do extra model building) so no significant effect (neutron stars similar).

Summary

- ‘Nuclear dark matter’ is an interesting possibility
- If properties of these states obey scaling laws, this can determine the number distribution from dark nucleosynthesis (attractor solutions)
- Possible to synthesise large-number ($> 10^6$) states, with either peaked (scaling) or power-law number distributions
- Potential consequences for direct detection, modifies relations to collider constraints, astrophysical signals
- Much remains to be done, e.g. specific models.

Backups

Toy Models

Many ways in which nuclear DM could be realised.

- Simplest: Fermion ψ mass M with Yukawa coupling $\lambda\overline{\psi}\psi\phi$ to scalar ϕ of mass m.
  Bound state of 2-nucleon system if $\alpha \geq 1.68\frac{m}{M}$. Natural size of m not too far from M. Putting together typically $\frac{m}{M} \geq 0.1$ so short range force
- SM with U(1) interactions turned off and small bare quark masses: i.e. SU(N) QCD (N=odd and N=even have interesting differences).
  'Pions' have freely adjustable PNGB masses $m_\pi \sim \sqrt{m_q f_\pi} \ll \Lambda$. Interactions are both derivative and non-derivative and spin-dependent. Also large number of meson resonances.